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A NON-COMPUTER APPROACH to the CRITICAL PATH METHOD for the CONSTRUCTION INDUSTRY

2nd Edition

JOHN W. FONDAHL
Associate Professor of Civil Engineering

PREPARED UNDER
RESEARCH CONTRACT NBy-17798
BUREAU OF YARDS AND DOCKS, U.S. NAVY

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STANFORD UNIVERSITY

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SUMMARY

The Critical Path Method for planning, scheduling, and control of project operations is a new and useful tool now becoming available to the construction industry. Successful applications have already proven its value. To date, except for the simplest of cases, the use of this technique has been largely dependent on programmed solutions by electronic computers. This report presents the mechanics of noncomputer methods for applying the Critical Path Method. These employ the same input data and furnish the same output information as the computer methods. In addition, alternate approaches to the solution of the problem are presented which permit the important scheduling variation phase to begin with a normal estimate rather than a set of data for an artificial condition.

There are three goals toward which this report is aimed. The first is to present a noncomputer method for obtaining the benefits of critical path scheduling that is practical to apply to many of the projects encountered by the construction contractor. The second is to develop the possibilities inherent in a step-by-step, manual solution to overcome some of the shortcomings of computer programmed solutions. The third is to offer the reader an opportunity to understand the details of the method and the assumptions upon which it is based, by discussing them and presenting a complete solution to an illustrative problem.

The solutions of a complex problem by computer methods, eliminating tedious calculations and possible errors, is a valuable step forward. This report is not intended to oppose such methods but, rather, to offer a stepping stone between conventional procedures and these more sophisticated practices. The need for such a stepping stone is justified on two bases. First, many potential users find it inconvenient to use electronic computers or are not yet "computer conscious." Second, the computer approaches are not completely satisfactory in all respects. It is anticipated that a broader acquaintance with, and use of, the Critical Path Method, made possible by

noncomputer methods, will lead eventually to an even wider employment of computer techniques as well as the development of improvements in them.

This report is not prepared for the casual reader. It attempts to present sufficiently detailed explanations of the procedures in as concise a manner as practical for the benefit of those who will actually apply these methods. Unless these details are followed closely and the effort is made to perform the calculations independently, using the examples given as a guide and check, the report will have limited value. Some of the procedures that seem complex upon initial examination will appear much simpler after a few trials are attempted. It should be remembered that although a project subdivided into a small number of operations is used to illustrate the mechanics of the methods, the procedures are designed with the intention that they may be applied to projects having a considerably larger number of operations.

INTRODUCTION

Planning, scheduling, and control of activities are functions required by most industries. A new and improved technique for performing these functions has been developed. It has been discussed under several titles including Critical Path Method (CPM),[1] Critical Path Scheduling (CPS),[2] Critical Path Analysis (CPA),[3] and Least Cost Estimating and Scheduling (LESS).[4] The use of the designation Critical Path Method, and the abbreviation CPM, has been adopted in this report. It appears to be the most widely used designation and does not imply a limitation in the use of the technique to only scheduling or estimating.

This report is concerned with the application of the Critical Path Method to the planning, scheduling, and control of construction projects by construction contractors. Most of the procedures discussed are as applicable to other industries, activities, and users as those chosen for this presentation. For example, maintenance jobs performed by an owner's own forces may receive similar analysis. However, the importance of the construction industry justifies the examination of any new tool for possible cost reduction from its standpoint alone. Moreover, a desire to be specific makes the selection of one potential user's problems a more satisfactory setting for discussion.

A construction project may be subdivided into a number of separate steps, or operations, necessary for its accomplishment. Each one of these operations may be performed by many possible combinations of methods, labor skills, crew sizes, equipment, and working hours. A major factor in choosing the best combination is cost and, on first glance, it would seem desirable to perform each operation at its least cost in order that the overall project may have the lowest total cost.

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[1] "Perini Corp. Pioneers CPM, New Tool for Job Management." Engineering News Record, January 26, 1961

[2] Sayer, J.S., Kelley, J.E., Jr., and Walker, M.R., "Critical Path Scheduling," Factory, July, 1960

[3] Clarke, R.W., "An Introduction to Critical Path Analysis." Graduate School of Business, Stanford University, March, 1961

[4] Backer, F., Jr., "A Discussion of Problems Involved in LESS (Least Cost Estimating and Scheduling)," IBM Applied Science, Dallas, Texas, 1960

In a broader sense, however, there are other costs associated with project completion. From the owner's standpoint, these include the costs or lost revenue to be saved by early completion and utilization of the facility being constructed. Such costs are sometimes anticipated in advance when liquidated-damage or bonus-penalty clauses are included in construction contracts. These factors form the basis for establishing completion dates that may make the performance of the project at least direct cost an impossibility. From the contractor's standpoint, ever increasing indirect or overhead costs, anticipated labor and material cost increases, and the early release of key personnel and equipment for other work, are factors that make project scheduling for lowest direct cost unattractive.

It is only by considering all the cost items connected with project performance and utilization that the best plan of action can be determined. This introduces time as an essential variable since almost all costs vary with time. But they vary with time in opposing directions. Direct costs tend to decrease as more time is permitted for performance. On the other hand, the costs associated with the duration of the project tend to increase as more time is taken for performance. It follows that there is some balance between direct cost and time that offers the best solution. For construction contracts, the owner attempts to find this balance point when he specifies the completion time, and the contractor attempts to find it when he plans his detailed schedule of operations.

The recognition of the joint and interrelated importance of both cost and time in project planning and in project control as well is not a new concept. For many years appraisals based upon good judgment assisted by limited trial and error solutions have been employed. However, systematic methods for considering the joint and interrelated effects of cost and time in reaching a schedule that gives lowest overall project cost is a new concept.

The problem is not a simple one. Each operation into which a project is divided can be performed in different ways at different costs and in different lengths of time. The problem would be simple if the overall project time had no importance. In this case, each operation would be performed in the manner that produced the least direct cost.

For reasons already discussed, this would be a rare situation. Overall project time must generally be reduced from that corresponding to least direct cost to arrive at a more favorable solution. This is accomplished by speeding up some of the operations. Speeding up an operation almost always results in higher direct costs since such measures as overtime or shift work, oversized crews with less effectiveness, larger equipment, and different, more expensive methods are generally required. To produce the same time decrease in one operation as another generally requires a different additional expenditure. The solution to the problem would still be relatively simple if the operations formed successive links in a single chain of events through the project. Then the project time could be decreased to the desired duration by progressively shortening those operations which could be altered at least increase in cost. However, this is also a rare situation. Instead of successive links in a single chain of events, there is normally a complex network of concurrent, overlapping and interrelated operations. Shortening a single operation may only serve to increase project cost without decreasing project time since other controlling operations have not been shortened. At the other extreme, shortening all operations simultaneously will increase project cost more than is necessary to obtain a corresponding decrease in project time. There are usually certain combinations of operations that may be shortened to produce the most economic project shortening. In other instances, shortening a single operation even though this is expensive may offer a better solution than shortening another operation that alone is less expensive but which requires concurrent shortening of other operations. Many such selections must generally be made in arriving at a project schedule. Without a systematic method on which to base these decisions, it is unlikely that the unique combination giving the best answer will be obtained.

The Critical Path Method offers this systematic approach. It also offers additional information for project control and other purposes beyond that provided by the more conventional project schedules as commonly represented by bar charts. Specific examples of the information provided by CPM and some of the resulting applications are as follows:

(1) It pinpoints the operations whose completion times are responsible for establishing the overall project duration. With these critical operations clearly identified, major attention may be directed toward keeping them on schedule in order that the planned completion date may be met.

(2) It gives a quantitative evaluation of the amount of leeway, or "float,"[1] that each of the other operations possess. Within the limits of float time, these non-critical operations may be started later or completed more slowly than the original schedule indicated, without detrimental effect on the project completion date. The importance of variations from the planned timetable of events is more clearly indicated to management if these limits of float are known. Moreover, boundaries are established within which operation times may be shifted to smooth out manpower or equipment requirements. This is desirable in the job planning stage.

(3) It shows the most economical scheduling for all operations for each possible project completion date. This is valuable planning information that permits correct consideration of both time and cost in choosing methods, equipment, materials, crews, and work hours. It replaces the trial and error juggling of operation times to arrive at a feasible solution which would seldom be the best possible solution.

(4) It provides the necessary data for choosing the best project completion date. The results from (3) may be plotted on a time-cost graph along with curves for indirect costs and bonus-penalty or liquidated damage payments. The sum of the curves would produce a total cost curve having a lowest cost point that would indicate the optimum project duration.

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[1] The term "float" has been widely adopted as a measurement of operation scheduling leeway.

(5) It offers a means for assessing the effect on the overall project of variations in one operation or the addition of an operation or operations. Change orders and extra work orders often affect the performance of more than the operations directly involved. Lack of a rational basis for determining this effect leads to frequent arguments and sometimes to inequitable settlements. The CPM analysis can indicate which other operations are affected and what the overall effect is, if any, on the project completion time. If the change would extend completion time but the owner desires that work be accelerated to meet an established completion date, CPM offers a means for rescheduling the operations still to be completed at the least increase in costs. Furthermore, such time and cost effects can be computed at any stage of contract completion.

The Critical Path Method has, in its short history, been successfully applied by a number of users in a variety of industries. These include both building and heavy construction. Many individuals and groups have worked, and are currently working, on its development and improvement. Its mathematical basis is an operations research procedure known as parametric linear programming.[1] The process is a complex one involving many computational steps. It is not surprising that the methods developed employ electronic computers. At least three of the major computer manufacturers have standard programs for solving various CPM problems, and they are working on others.

The work on which this report is based involved an independent effort to develop the mechanics necessary for applying the Critical Path Method. It was also aimed at the application of these mechanics by "manual" methods. While there are many disadvantages to employing a complex method without the aid of an electronic computer, there are also many justifications for a manual procedure. These justifications are possibly temporary ones and may disappear when computers become a

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[1] Kelley, J.E. Jr., "Critical-Path Planning and Scheduling: Mathematical Basis," Operations Research, Vol. 9, No. 3, May-June, 1961.

more common tool in contractor's offices and better programs are devised that can more satisfactorily handle contractor's problems. It is quite possible that the use of CPM based on computer methods will eventually be almost universally adopted by construction contractors. Because such a movement is already underway, it might seem a disservice to recommend a noncomputer method, since the elimination of routine and tedious calculations by computers serves to release more time for the application of judgment and to decrease the probability of errors. However, in this instance, introduction of noncomputer methods is not considered a step backward. Rather, it is a desirable intermediate step between those methods almost universally in current use and the more sophisticated ones that are becoming available. The need for such a stepping stone can be justified by the two conditions, already mentioned, described in more detail as follows:

(1) <u>Limited current use of computers in the industry</u>:

Although a number of the larger organizations have employed specialists and established computer centers, most smaller companies and most field organizations do not have such facilities. It is usually true that they could make use of computer service centers and, in many cases, home office facilities. However, this is either inconvenient or there is a lack of appreciation of the benefits that are available. A manual approach might encourage a much more widespread application of CPM for problems of moderate complexity involving original project planning, subsequent revisions to project planning, project control, and contract change settlements. As familiarity with the methods grows and more complex problems are undertaken, the use of computers would naturally become a logical step to take.

(2) <u>Shortcomings of computer programs developed to date in offering a completely satisfactory approach to the problem</u>:

All methods for solving the problem of schedule variations require simplifying approximations for the operation time-cost relationships. But the computer methods require that these approximations apply over wider ranges. Moreover, they are much less flexible in allowing revisions of the input data as the schedule takes shape. A step-by-step manual method allows the planner to retain more judgment control in making changes in the input data. It also permits him to pick a starting point that limits the range over which approximations must apply. With the computer, all the input data must be supplied at the outset, while with the manual approach the introduction of new data is allowed as the schedule develops. The computer approach assumes that each operation is independent except for the sequential relationships indicated by the project network. Actually, cost interactions between operations frequently result when schedule changes are made. The manual approach permits the planner to experiment with such changes that may affect several operations rather than just one.

As more experience in applying CPM mechanics to construction problems is developed, improved computer programs that remedy such shortcomings will probably be developed.

Against these justifications for a manual approach to the problem, there are some very obvious disadvantages, including:

(1) A considerable amount of computational and data recording effort, even for problems of little complexity.

(2) The favorable opportunity for error involved in any long, routine set of calculations.

(3) The much longer time to perform the computational work.

(4) The reluctance to make changes in the project network that would require much of the computational work to be repeated.

(5) The inconvenience of frequently updating the schedule which also may require repeat performance of a considerable portion of the computational work.

To some extent these disadvantages may be lessened by such measures as:

(1) Developing methods for checking computed results.

(2) Developing and using flexible, standard printed forms on which to perform computational steps.

(3) Developing procedures for easy updating of data. These might include:

    (a) Data presentation on peg boards
    (b) Data presentation on chalk boards
    (c) Data arrangement by card file to permit simple changes in precedence order

(4) Carrying through only an incomplete but sufficient solution of the problem. Considerable work is involved in performing the schedule variation phase of the method. While this procedure may be worked through many cycles, each cycle represents an improved schedule, and only a few cycles may give the optimum schedule. Therefore the entire process usually need not be carried to completion.

(5) Utilizing a well conceived, conventionally estimated schedule as a starting point. The computer programs usually start at the extreme end of the overall project cost-time curve and proceed to shorten the project from that point. Starting with a feasible schedule for the contract time allotted, or considered reasonable, a very few cycles of calculations may be required to obtain the optimum schedule.

Before considering the manual method to be presented, a few remarks on the history of the development of the Critical Path Method are in order.

HISTORICAL DEVELOPMENT

The basic concepts and detailed procedures for the Critical Path Method have been developed since 1956. Two pioneering efforts[1] resulted in two parallel, but different, approaches. These two can be appropriately designated as the "probabilistic" approach and the "deterministic" approach. The probabilistic method first appeared as a contract control system known as PERT (Program Evaluation Research Task). It was developed under the sponsorship of the Special Projects Office, Bureau of Ordnance, U.S. Navy, for use as a control tool over contracts for the Fleet Ballistic Missile Program. The various contracts included in such a program correspond to the operations making up a single construction project. These contracts involved, to a large extent, research and development work and the manufacture of component parts that had never before been built. Hence neither time nor cost could be estimated accurately. Instead, completion times were treated on a probabilistic basis. Contractors were asked to predict an optimistic, a most likely, and a pessimistic estimate of time requirements. A formula was devised for weighing these estimates to determine an expected time of completion for each contract. These times were then incorporated into a control system. This system did not consider cost as a variable. Other more recent work has been reported,[2] known as PERTCO (PERT with costs), to include cost data on the same sort of probabilistic basis. The PERT system has been adopted by the Air Force and its name modified to PEP (Program Evaluation Procedure). It is being used by the Air Force and the Army in connection with various missile programs.

The deterministic case is much more suitable for construction applications than is the probabilistic case, since it is used when cost as well as time is to be considered as a controlling variable. It

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[1] "Better Plans Come From Study of Anatomy of an Engineering Job," Business Week, March 21, 1959.

[2] Chipman, J.S., "PERT With Costs," Technical Report 112 SRP, WSPACS Working Paper No. 4, Aerojet General Corporation, February 15, 1961.

requires that both cost and time be estimated with reasonable accuracy. The pioneering effort for this case was accomplished by a team of engineers and mathematicians representing the E. I. duPont de Nemours and Co., Inc. and the Sperry-Rand Corporation. Their method was designated as a Project Planning and Scheduling System[1] and was successfully applied to a number of large and complex jobs involving design, construction, and maintenance work. Two of the individuals largely responsible for this effort later joined the firm of Mauchly Associates, Inc., Ambler, Pennsylvania. The Mauchly organization has offered numerous workshop courses in many locations in the United States and Canada to acquaint industry with the application of CPM. Construction companies have been included in the groups participating in these workshops and, in turn, have applied the techniques to their own problems.

Other work has been in progress by the Remington Rand, General Electric, and IBM computer organizations. Each of them has developed standard programs for the solution of certain scheduling problems and is working on more advanced problems. Several other organizations are also working on new or alternate methods. These include Lockheed, Aerojet-General, and, in the construction field, the Bechtel Corporation and Kaiser Engineers. Other construction companies, including Utah Construction and Mining, Fruin-Colnon, Perini Corporation, and Peter Kiewit, are already applying CPM to their operations.

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[1] Walker, M.R. and Sayer, J.S., "Project Planning and Scheduling," Report 6959, E.I. duPont de Nemours and Co., Inc., Wilmington, Delaware, March 1959.

THE CPM APPROACH - GENERAL

The Critical Path Method may be subdivided into three distinct phases. Phase I is the construction of a representation, or network model, of the sequential relationships of the operations making up the project. This phase is a prerequisite for each of the other two phases. It has been classified as a separate phase because its accomplishment provides a useful product even if no further steps are taken. The chart produced offers a better understanding than a conventional bar chart of those relationships that control the order of performance of the various operations. This understanding is valuable both in project planning and control. Computer solutions differ little from a manual solution insofar as this phase is concerned. The network formulation is essentially a long-hand chore. However, computer programs have been developed for the proper numbering of the network elements and their renumbering when, subsequently, operations are added or deleted.

Phase II develops information primarily useful for control purposes. From it, a construction schedule may be presented that gives more information than a conventional bar chart. This is the phase that determines which operations are critical, i.e., which determine the overall project duration, and how much float the remaining operations have. While this type of information is most directly useful in project control, it also can have an important use in planning. Knowledge of operation floats can be used as a basis for schedule shifts to smooth out labor and equipment requirements. Considerable current effort is underway to devise computer programs that will perform this shifting in the optimum fashion to achieve manpower leveling in accordance with stated restrictions. Such programs might simultaneously consider many different labor classification (or different pieces of equipment). Phase II information can be developed, even for relatively complex projects, by manual methods within very reasonable limits of effort. The computer method's chief advantages are in the elimination of errors and in the ability to quickly revise the results if input information is altered. Probably the great bulk of applications of Critical Path Scheduling in the construction industry to date have gone only as far as Phase II. The fact that Phase I must be accomplished manually and that Phase II may be accomplished manually without unreasonable effort, indicates that for these two phases alone, noncomputer methods offer invaluable aid to the industry.

Phase III introduces the possibility of performance time variations and also introduces cost data. The objective is to determine the operation scheduling that produces the least direct cost for a given project duration or to determine both the project duration and corresponding operation scheduling that produces the least overall project cost. This phase is the most difficult one to solve by any method. It is the one that has been least applied. However, it is the phase that offers the greatest possibilities for cost reduction. It is felt that the manual method to be presented in this report can be used to solve problems having a moderate degree of complexity. Moreover it is felt, as indicated earlier, that there are certain advantages to be gained by working such problems manually at the present stage of CPM practice.

Phase II not only precedes Phase III but also follows it. After optimum durations for each operation have been determined by application of Phase III, then the corresponding new start and finish dates and operation floats would be determined by Phase II.

To illustrate the procedures presented in this report, a project that has been divided into a number of operations will be used as an example. While this project will have only 18 operations, the time and network relationships will make it complex for its size. The factors encountered in a much larger problem will be present. As the project is shortened, a total of six different critical paths through the network will be developed. The entire process of shortening the project, (Phase III), starting at the point of least project direct cost and proceeding to the all-crash, maximum direct cost point, will be discussed in some detail. This will serve to acquaint the user with the complete mechanics of the system. It will also duplicate the full scope of information developed by the computer approaches. A less detailed discussion will then be presented to indicate how the Phase III information might be developed by working in the opposite direction from a beginning at the all-crash end of the project time-cost curve. Finally, these two approaches will be combined to offer a third approach. This approach commences with a conventionally determined schedule and cost estimate and applies Phase III mechanics to improve this schedule until the optimum schedule is reached.

PHASE I

To develop any sort of detailed schedule, a project must first be broken down into the operations necessary for its completion. A representation of this breakdown on paper may be called a "model". When this model indicates the sequential relationships of these operations, it assumes the appearance of a "network". Hence the term "network model" will be used to describe the chart showing the project breakdown and sequential relationships between the resulting operations. Proper visualization and construction of this network model is the most basic and important step and is probably the most difficult part of the Critical Path Method. This is as true for computer-oriented procedures as for manual procedures. Network formulation is essentially a longhand job in both cases.

The degree of project breakdown can be varied considerably, but certain factors often affect the divisions made. These factors include the following:

(1) The type of work and predominant labor classification required for its performance. Operations performed by different types of crews, such as electrical, dirt moving, or concrete are generally separated.

(2) The structural elements involved. In a building, column construction may be grouped separately from wall construction or floor slab construction.

(3) Responsibility for the work. Operations of various subcontractors are separated from those of the general contractor.

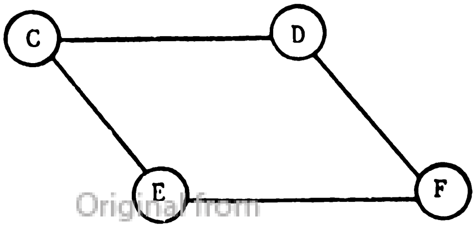
(4) Location of the work. Work performed at different times or by different crews because it is at different locations in the structure may be considered separately.

(5) Owner's breakdown for bidding or payment purposes. On a unit-price job, or a lump sum job where a breakdown for progress payment purposes has been made, it is often desired and sometimes required that the construction

schedule include the same breakdown.

The operational breakdown for formulation of the CPM Phase I network also depends upon a careful definition of the sequential relationships among the operations. Some operations must precede others, some must follow others, and some may be performed concurrently with others or independently of them. But every operation in the network must have a definite event to mark its possible commencement. This event may either be the start of the project or the completion of some other operation which directly precedes it. Since the completion of an operation must signal the start of some other related operation, it is not possible to have overlapping, related operations as are frequently indicated on conventional bar charts. Where such a condition exists, the operations must be divided further. The earlier work must be so subdivided that the portion that must be completed before later work is commenced is made a separate operation. And the later work must also be subdivided, if necessary, so that just that portion made possible by completion of the preceding operation is included in a separate operation. To more clearly illustrate these requirements, consider a project involving a concrete wall. Let the operation of constructing the footing for the wall be represented by A. Let the operation of constructing the wall be represented by B. A portion of the project network could be represented, then, as follows:

This indicates that Operation B follows the completion of Operation A. In other words, the entire footing is built before the wall is built. Suppose, however, that the wall is long and that after the first half is built, the wall overlying the completed portion can be started. A conventional bar chart would indicate this condition by overlapping the bars representing the two operations. However, the CPM network would require dividing Operation A into two new operations, C and D, and Operation B into two new operations, E and F. The equivalent portion of the network would be represented as:



This diagram states graphically that after the first half of the footing is built (Operation C), then the first half of the wall may be built (Operation E), and the remainder of the footing may also be built(Operation D). Note that it was also necessary to subdivide former Operation B, since the construction of the second half of the wall (Operation F) could not follow directly after the new Operation C. Care must be exercised in drawing the connecting lines between operations in the project network. For example, the foregoing figure states that Operation F, the building of the second half of the wall, not only requires completion of the footing but also requires completion of the first half of the wall. This might be the case if, for example, the same forms were to be used as on the first half of the wall. However, if building the second half of the wall depended only on completion of the footing, the line between E and F should be deleted.

These operational subdivision requirements tend to make the project breakdown for CPM more complex than for conventional scheduling. However, because the planner is forced to pinpoint more precisely the operational relationships, a more valuable representation of the job is obtained in the resulting network chart. It was mentioned that the degree of breakdown is variable. The operations chosen may represent relatively large segments of the overall project or very detailed steps. For example, the building of both footing and wall might have been lumped together in the preceding illustration. On the other hand, the work might have been broken down into considerably more detail to include such operations as: build forms, erect forms, place reinforcing steel, place concrete, cure, strip forms, etc.

The project network may be represented in at least two ways. The method generally used, and that on which the available computer programs are based, involves an arrow notation. The method on which this report is based involves the circle and connecting line notation just illustrated in the wall and footing example. One system of representation is easily convertible to the other. The circle notation has been devised for the manual method because it simplifies the mechanics of the Phase III procedure. It also simplifies construction of the network model. However, the arrow notation will be discussed briefly since it is widely used. Moreover, since a different and useful form of progress schedule can be presented in a modified version of the arrow chart, the use of both systems might be desirable for those using the manual method.

In the circle and connecting line notation, each operation is represented by a circled number and its relationships to those operations that either must directly precede or follow it are shown by connecting lines to their circled numbers. The position of the circles in the network representation is free to vary in the vertical direction. It is restricted in the horizontal direction by the requirement that the horizontal projection of a connecting line to any following operation must run from left to right. One other restriction on horizontal positioning, though not essential, produces a more orderly arrangement that is convenient for data groupings in Phase III. A length for the shortest horizontal projection of a connecting line between two operations is chosen and is called a "sequence step". The optional requirement, then, is that the circle for every operation will be positioned horizontally the least possible number of sequence steps from the left border of the network chart.

The first step in constructing a network model is to subdivide the project into operations. Then a rough diagram is commenced. (see Fig.1) There is no attempt to number the operations at this point. Rather, each is represented by a circle with a descriptive title written next to it. As each operation is entered on the worksheet, the questions are asked:

(1) What operations must be completed to make this operation possible?

(2) Can this entire operation be performed upon completion of the preceding operation?

Answers to these two questions will often indicate that the prerequisite operation must be broken down further or that the operation being entered must be broken down further. Sometimes new operations are required. For example, procurement of materials or delivery of equipment that must be ordered in advance should be included as operations in the network.

Connecting lines are drawn between operations as the network is developed to indicate which operation must directly precede or follow other operations. On this rough diagram the horizontal projection of a connecting line from a preceding to a following operation may run in either direction. It is therefore necessary to use arrowheads temporarily to indicate the direction. When the rough network has been completed, if it is desired to use the sequence step method for a more orderly final arrangement, the sequence step of each operation can be determined. The initial operations are given a sequence step number of zero. Each other operation is given a sequence step number one greater than the highest sequence step number of the operations directly preceding it.

The final network is drawn from the rough diagram. (see Fig. 2). Each operation, indicated by a circle, is located horizontally to the right of operations preceding it. Preferably, each operation is located horizontally at its proper sequence step. Operations are located vertically in such a position that connecting lines may be represented clearly and, if practical, as straight lines. All connecting lines are drawn but arrows heads are no longer necessary. The brief description of each operation is entered next to the appropriate circle. Then all operations are numbered by inserting the number inside the operation circle. Numbering is commenced at the upper left corner of the network and precedes vertically downward for all operations in the same column or sequence step. Then numbering is carried to the top of the next column from where it precedes downward in that column. This procedure continues until the extreme right border of the network has been reached and all operations have been numbered. The essential requirement for the numbering system is that no operation will follow another operation of higher number. The sequence step system is helpful in drawing and numbering the final network. Once the maximum number of sequence steps has been determined on the rough diagram, a sequence step can be assigned a horizontal dimension that will make the network the desired length for its final portrayal.

Figure 1 shows a rough diagram for a simplified version of a small pier job. The operations have not been numbered at this point. No special attempt was made to draw connecting lines straight. Their directions are indicated by arrowheads. Sequence step numbers, the small numbers beside the circles, were determined upon completion of the network. Figure 2 shows a final representation of this same network. The operations have been plotted according to sequence step and have been numbered. The operation descriptions are included for information purposes but all further computational work would be based on the assigned operation numbers. Figure 3 shows another and somewhat similar project network. Although containing fewer operations, for convenience in presenting the various worksheets that follow, its interrelationships and data offer a considerably more complex problem than that for the project of Figure 2.

The alternate, and more common, method of network representation is the arrow diagram. In this method, each operation is represented by an arrow whose tail is positioned at the point where the heads of arrows representing directly preceding operations meet. Its head, in turn, determines the point where the tails of arrows representing directly following operations are to be positioned. Ordinarily, the arrow is not a "vector" since its length and direction have no significance. However, the final network chart is drawn in such a manner that the horizontal projection of each arrow, from tail to head, is a line running from left to right. Moreover, for purposes of a new type of progress chart, the length of each arrow may be chosen to make its horizontal projection correspond to a selected time scale.

The arrow network is labeled to give the tail and head of every arrow a number. The number of the head must always be greater than the number of the tail. Arrows meeting and departing from a common point have one number in common. Each arrow, and hence, each operation, has two numbers that define it. The first number is that appearing at the arrow tail and the second number is that appearing at the arrow head.

This method has the advantage, for computer applications, that the resulting operation numbers indicate sequential relationships. For example, every operation having a second number of 8 directly precedes all operations with first numbers of 8. However, sequential relationships in the manual method are determined by visual observation of the network model. Therefore the simpler single numbering system is satisfactory. The arrow notation method has a disadvantage in the requirement that all arrowheads of immediately preceding operations meet at a common point and that all arrowtails of immediately following operations commence at this common point. This is sometimes impossible because a group of preceding operations may have one common following operation but other following operations may not be common to the group, or vice versa. This diagramming problem is overcome by introducing "dummy" operations having zero time and cost.

Figure 4 is an arrow representation of the same network shown with circle notation in Figure 3. As an example of the difference in numerical designations of the same operation by the two systems, Operation 9 of Figure 3 becomes Operation 4-8 in Figure 4. (The operation numbers from Figure 3 have been shown in Figure 4 in lieu of operation titles.) With the arrow notation network it was necessary to add nine dummy operations for this example. As an example to illustrate the necessity for one of these dummy operations, consider first a relationship shown in Figure 3. It is indicated there that Operations 8, 9, and 10 all follow Operation 6 and that Operation 10 is preceded by Operation 2. When we translate these sequential relationships to arrow notation on Figure 4, we have no trouble in showing Operation 8 (now Op. 4-5) or Operation 9 (now Op. 4-8) following Operation 6 (now Op. 2-4). But we cannot show Operation 10 starting at the arrowhead of Operation 6, since Operation 10 also follows Operation 2, but Operation 2 does not precede either Operation 8 or Operation 9. The problem is solved by introducing dummy Operation D_2, designated as Operation 4-6. All network relationships are satisfied now because the dummy operation has zero elapsed time. Operation 10 (now Op. 6-7) still follows Operation 6 directly even though their arrows have been physically separated on the diagram by a dummy arrow.

The preceding paragraphs have discussed the arrow diagram and indicated how it is constructed and how a circle diagram may be translated to a corresponding arrow diagram. The circle notation will be employed in presenting this report, except that the arrow diagram will be referred to again in the discussion of Phase II, where its application to progress charts is considered.

PHASE II

The project now has been subdivided into operations and a network chart prepared that shows all the operation sequencing interrelationships. The only further information required for Phase II scheduling work is a time estimate for each operation. These estimated times usually stem from one of three situations, as follows:

(1) If the CPM technique is only to be applied through Phases I and II then the estimated times represent some feasible combination of operation times that permits completion of the project in the allotted or desired time. These operation times are determined by conventional estimating and scheduling procedures.

(2) If Phase III of the CPM technique is also to be applied, then the estimated operation times are those that give the project duration and cost that serves as the starting point for the Phase III computations. If Phase III computations follow the usual pattern for computer solutions, this starting point would be the schedule that gives the least direct project cost. The time for each operation would be that at which it could be performed at least direct cost. These times and costs are commonly referred to as "normal time" and "normal cost."

With the manual method Phase III computations also can be started at the project "all-crash" condition. Then each operation would be scheduled at the shortest possible, or crash, time. Later it will also be shown how Phase III computations may commence with a conventional estimate. Then the operation times would be the same as described in (1) above.

(3) After Phase III of the CPM technique has been completed, or carried as far as the planner wishes to take it, the resulting revised operation times may then be fed back into Phase II.

With estimated times available for every operation, the computation work of Phase II is performed on one worksheet. It is shown as Figure 5. To simplify following the calculations, selected columns of figures are presented on four separate sheets. For actual computation work, a single sheet would suffice. The eighteen operations are listed by number and the estimated time of each is given in days. Since some of the results will subsequently be used to illustrate Phase III computations starting at the project least cost point, the operation times chosen are the normal, or least cost, times. The procedures are exactly the same regardless of which basis is chosen for operation times.

In job scheduling there is another choice to be made in connection with the time basis. Time may be in working days or it may be in calender days. The times given in the computations in this report will be assumed to be in working days unless otherwise indicated. Care must be exercised to be consistent. Such items as material or equipment deliveries are generally quoted in calendar days, and the corresponding operation times must be converted to working days if other operation times are in working days.

To perform Phase II computations Figures 3 and 5 must be used together. Figure 5A shows the first step. For every operation, the earliest possible starting date is determined. To this is added the operation duration time to obtain the earliest possible finish date for the operation. The network chart, Figure 3, shows that Operations 1, 2, 3, and 4 may all start when the decision has been reached to begin the project. Therefore zeros are entered for their earliest start time. "Time zero" is the end of the day before construction begins or actually the beginning of the first day. Operation 1, which requires 5 working days, has its earliest possible finish at time 5. This may similarly be interpreted to mean the end of the fifth working day or the beginning of the sixth day. All other earliest start and finish dates may be obtained from those of preceding operations. For example, the network indicates that Operation 5 follows Operation 1. Therefore the earliest start date for Operation 5 is equal to the earliest finish date of Operation 1. If an operation follows more than one other operation its earliest start date is equal to the latest of the "earliest finish" dates of the preceding operation. For example, Operation 12 follows

Operations 5, 9, and 10. The network relations require that all three of these operations must be completed before Operation 12 may be commenced. Their earliest completion dates are, respectively, 17, 15, and 25. Therefore, the earliest start date for Operation 12 is 25. If operations are numbered by sequence step groups as described in Phase I, it will always be possible to run directly down the list of operations in numerical order and determine earliest start and finish dates from preceding data. The earliest finish date for the final operation is the total project time. In this example the earliest finish of the final operation, Operation 18, is the end of the 64th day. This means that the project could be completed in 64 working days.

The next step of Phase II is to fill in the columns for the latest start and finish dates for each operation. Figure 5B illustrates these computations which are quite similar to, but the reverse of, those of Figure 5A. A starting point is determined by using the earliest finish time of the final operation as the latest finish time for the same operation. Then the latest start time is obtained by subtracting the operation duration time. From the bottom of the columns, computations are carried upward. For each operation the latest finish time is equal to the latest start of the following operation. For example, the latest finish time of Operation 16 is equal to 61, the latest start time of Operation 18. Where an operation precedes more than one other operation its latest finish time is equal to the earliest of the "latest start" times of the following operations. For example, the network indicates that Operation 10 precedes Operations 12, 13, and 14. The latest start times of those operations have already been determined to be, respectively, 35, 30, and 28. Therefore, the latest finish time of Operation 10 is 28.

Having determined both the earliest and latest possible finish dates for each operation, it is possible to determine quantities known as "Total floats." Total float, as shown on Figure 5C, is the difference between the earliest finish time and the latest finish time. The same results would be obtained by subtracting the earliest start time from the latest start time. Float is a measure of the time leeway available for an operation. The total float figure states the number of days by which the finish date of an operation can exceed the earliest possible finish date without affecting the duration of the overall project.

A zero total float indicates that an operation has no leeway and, therefore, is one of the operations that establishs the project duration. If its finish date were later, the project finish date would be later by the same amount. Such operations are labeled "critical operations." There must be at least one chain of critical operations running from beginning to end of the project. Its total time gives the total project duration time. There may be more than a single chain. If so, these may be determined by visual observation of the network chart. To aid in this process, those operations for which a zero total float has been calculated may be double circled, or circled in color, on the project network chart. Connecting lines between doubled circled operation numbers may be doubled or traced in color. The resulting chains of operations, or "critical paths," will be evident by observation. In this example, there is a single critical path consisting of Operations 3, 13, 15, 17, and 18. The sum of the operations times for these five operations determines the project duration of 64 days. Should any of these operations take more than the estimated time or be delayed at all, the project time will be lengthened by a corresponding time. All of the other operations have some degree of leeway. Any one of these may be performed more slowly or have elapsed time between its start and the finish of a preceding operation, as long as the increase falls within the limits given by its total float. Within these limits, the schedule change will not affect the 64-working-day project duration.

Additional scheduling information, known as "free float" time, can also be obtained for each operation as indicated on Figure 5D. The free float of an operation is the difference in days between its earliest finish date and the earliest of the "earliest start" dates of all of its directly following operations. With the aid of project network chart observations the free floats can be calculated from data on Figure 5D. For example, Operation 4 is shown on the project network to be followed only by Operation 14. The earliest finish date of Operation 4 is 20 and the earliest start date of Operation 14 is 25. Therefore, Operation 4 has a free float of 5 days. As a second example, consider Operation 6 which is followed by Operations 8, 9, and 10.

The earliest finish of Operation 6 is 11. The earliest starts of the three following operations are, respectively, 11, 11, and 15. The earliest of these is the same as the earliest finish of Operation 6 which, therefore, has zero free float. The free float figure states the number of days by which the finish date of an operation can exceed the earliest possible finish date without affecting any other operation.

The concepts of "total" and "free" float can be illustrated graphically by an adaptation of the common bar chart. Figure 6 shows such a construction schedule for the example worked. The solid black bars show operation durations and earliest start and finish dates. These alone would be the equivalent of a conventional bar chart for the same operational breakdown. Where there is no extension of the solid black bar, there is no float, and the operation is a critical one. Extensions of the bars represent the amounts of total float. However, the labels "Interfering Float" and "Free Float" have been assigned instead. Free float, as previously explained, is the time range over which variations in operation completion time may occur without having any effect on either project completion date or the amount of float of other operations. The difference between total float and free float is called interfering float to indicate that, while operation completion in this time range does not affect project completion time, it does affect some subsequent operations by decreasing their floats. If there is zero free float, then interfering float equals total float. This bar chart can be constructed to also show calendar days, to provide space for actual progress bars, and to indicate periodic percentage completion estimates, as is done with conventional progress charts. Of greater importance, however, is the additional information for control purposes. When operation performance fails to coincide with the conventional solid black bars of the chart, management can better evaluate the importance of the deviation. The chart also indicates possible scheduling shifts for planning purposes that will smooth out labor and equipment requirements or improve financial demands. For example, any operation can be intentionally shifted within its free float range with no adverse effect on the remainder of the schedule. Beyond that, operations can be intentionally shifted within their interfering float range with a determinable effect on the amount of leeway of later operations.

Another form of construction schedule chart, utilizing the arrow notation, is in use by industry. Figure 7 shows such a chart for the example under discussion. Figure 7 is the network chart of Figure 4 with two modifications. First, the arrows, except those for dummy operations, are drawn to such length that their horizontal projections conform to a time scale given both in working days and calendar days. Secondly, the arrows representing the critical path for the network are plotted in a straight line, horizontally through the center of the network, to give added emphasis. (Because dummy Operation D6 is required between critical Operations 13 and 15 this network requires an offset in the straight line) Free floats are indicated by extending the operation arrows to the appropriate junction point with a wavy line. This type of chart has the advantage that it retains the network chart representation and so clearly indicates the relationships between operations. It has the disadvantage that it does not indicate directly the amounts of total or interfering float.

In summary, Phase II of CPM produces a valuable end product for planning as well as control purposes. It may be applied to data available from ordinary estimating procedures provided that the project breakdown is made in a way that permits construction of the Phase I network chart. Once the project network has been established and the operation duration times estimated, Phase II calculations are purely mechanical. There are no opportunities nor needs for judgment decisions during the process. If a computer is available, this is an ideal use for it. The results will be obtained very rapidly, and there will be fewer errors. It is conceivable that the computer might be programmed to print out a bar chart directly, with all the additional information available from Phase II shown on it. If modifications are made in either the project network or in the estimated duration times, the computer can produce a revised schedule rapidly. On the other hand, because the amount of effort required for a manual solution is not prohibitive it should be clear that Phase II of CPM will make a valuable planning and control tool even when the services of electronic computers are not readily available.

PHASE III - GENERAL

Phase I of CPM provides a graphical representation of the sequential relationships between the operations that make up a construction project. Phase II provides valuable scheduling information for a single project duration. Neither of these phases introduce costs and, therefore, time-cost relationships have not been considered. In the Introduction to this report considerable emphasis was placed on the importance of a systematic method for taking into account time-cost relationships and arriving at the most favorable balance between them. Phase III does this and, for that reason, is the heart of the Critical Path Method. The solution to CPM Phase III provides the information for plotting a project time-cost curve and for obtaining a complete set of operation lengths to correspond with all points on that curve. The costs given by this curve are the lowest obtainable total direct costs at the corresponding project duration times.

Since Phase III deals with time-cost variations, a starting point from which to make these variations is the first prerequisite. One point on the project time-cost curve that may be determined in advance, with the aid of Phase II calculations, is that for least direct cost. This is the starting point for computer approaches. The only other point on the curve determinable in advance is that for "all-crash" performance. It may serve as an alternate starting point. This report will propose as a new starting point one of an infinite number of possible points that lie above the ideal time-cost curve. These points are determined by the project cost and duration as determined by a conventional estimate.

To establish the "least direct cost" starting point, each operation is performed in the manner resulting in least direct cost. The sum of the least costs for all of the operations gives the least possible project direct cost. The corresponding operation durations can be used as input data for Phase II and calculations can be performed as shown on Figure 5A to determine the project duration that corresponds to least project cost. These times and costs, for both operations and the overall project, are designated as "normal time" and "normal cost." Referring to Figure 8, point A represents a project normal time and cost point. In the next section of this report, the mechanics for developing the curve AC, starting at point A, will be given.

Most operations can be speeded up from the normal time required for performance at normal, or least, cost. This speeding up adds to the costs as it represents such steps as overtime work, multiple-shift operations, larger but less efficient crews, purchase of materials or services at higher prices, and the use of more expensive equipment and methods. Each operation has some practical time limit beyond which it cannot be shortened further. This is referred to as the "crash" time and the corresponding cost is the "crash" cost. If the estimator provides crash time and crash cost data for each operation, a second point on the project time-cost curve may be located. This is the "all-crash" point represented by point B on Figure 8. This represents the shortest possible project time and may be obtained by using the operation crash times to obtain the earliest finish date of the final operation by the procedure of Figure 5A. The cost is the summation of the crash costs of every operation and, therefore, is the highest possible cost. It would be, for example, the cost to a contractor who, faced with a crash type job, embarks on an across-the-board speed-up of all operations. However, generally the project crash time can be achieved at a cost much lower than the all-crash cost. Many operations on the project crash time schedule are not critical; i.e., they do not affect project duration. Hence, all that is accomplished by crashing such operations is to increase project cost without decreasing project time. The point on the project time-cost curve that corresponds to the lowest cost for crash-time performance, represented by point C on Figure 8, cannot be determined until Phase III computations are performed. A later section will give the mechanics for developing the curve BCA, starting at point B.

On Figure 8, point A represents the least project cost for the longest project time to be considered. Point C, undetermined as yet, represents the least project cost for the shortest possible project time. There exists between these two points an entire range of possible project durations. For each such duration there exists a schedule of operations that gives the least project cost. Curve AC represents these optimum costs. To obtain curve AC, beginning at A, the procedure involves shortening critical operations that reduce the project time at the least increase in project cost. This shortening is accomplished in steps using in turn those operations or combinations of operations that have progressively larger ratios of cost incurred to time saved. The curve from A to C is not a smooth one as shown in Figure 8. Rather, it is a series of straight line segments with increasingly steeper slopes. The result is a concave upward curve that indicates that each new time reduction comes at a higher cost.

Point C is reached when no further project shortening can be accomplished. This occurs when all the operations in one critical path have been shortened to their crash limits. There may be operations in other critical paths which can be shortened further, but it will be found that such shortening makes them non-critical and does not affect project duration. The critical operations at point C include not only the original critical operations for the normal project schedule but, generally, a number of additional operations that have become critical as the project time has been shortened. As indicated above, nothing is gained after point C is reached. Non-critical operations, and critical operations that would become non-critical upon further shortening, may still be shortened. However, this will not reduce project time further, but will increase project cost as indicated by CB on the curve.

Another procedure for developing the project time-cost curve may commence at the second of the two obtainable starting points on the curve, point B. Briefly, this procedure requires first lengthening all non-critical operations that are initially being performed at crash rates. This lengthening permits cost decreases without lengthening project time, as indicated by the vertical line from B to C. After all non-critical operations have been lengthened either to their normal time limit or until they have become critical operations, further project cost reduction is achieved by lengthening critical operations and, hence, project time. In this case the operations, or combinations of operations, that produce the greatest decrease in cost per unit increase in time are first selected. A segmented curve from point C to point A with progressively flatter negative slopes results. It would trace in reverse the curve from A to C obtained by the previously described procedure. Point A is reached when all operations have been lengthened to their normal times.

There are two reasons for developing this second alternate procedure. The first is that the all-crash point of the project time-cost curve may be considerably closer than the normal point to the desired solution. This would be the case when an owner has established a contract completion date that obviously requires a near-crash effort to meet. Since Phase III computations are often terminated when an optimum schedule for the specified project duration has been obtained, it is advantageous to start at the nearest point to this specified time. Secondly, the mechanics of the procedure for moving to the right along the project time-cost curve have been developed because ability to move in either direction is required by the third procedure.

Certain other aspects of project scheduling variations can be shown by Figure 8. An approximate reverse image of curve ACB can be produced as curve ADB if, starting at point A, the least advantageous scheduling changes are progressively made, rather than the most advantageous ones. For example, if, first, all the noncritical operations were crashed and then changes in combinations of critical operations producing the greatest cost increase per day of project shortening were made, this curve would result. The area enclosed by ACBD includes an infinite number of possible scheduling solutions for the project. For any given project duration a scheduling solution represented by the point on curve DB is the worst possible solution. And the scheduling solution represented by the point on curve AC is the best possible solution. In between these are many, many, possible solutions of varying degrees of effectiveness, any of which may be improved. A conventional estimate and schedule would probably result in such a scheduling solution. Depending on the skill and judgment of the estimator, the schedule might be represented by a point very close to curve AC or some distance from it. Since this distance, graphically and actually, represents dollars, a procedure to improve such a schedule is of value. Once the best schedule for a given project duration is determined, then further experimentation may be made in varying the project duration by moving either way along curve AC.

This discussion indicates a third possible starting point for the Phase III procedure. This is represented by point E on Figure 8 and corresponds to the schedule and cost data obtained from a conventional estimate. The first step is to lengthen any noncritical operations that can be performed more economically with additional time. Curve portion EF represents the cost improvement obtained in this manner. Then to reach the ideal curve, whose position is as yet unknown to the planner, a "wiggling-in" process must be employed. Alternately, the project is lengthened over one cycle of computations by using the combination of critical operations producing the steepest rate of cost decrease and then shortened back to the original duration by using the combination of critical operations that produces the least rate of cost increase. The lengthening slope and the return slope tend to approach one another as the process continues until a point is reached when they are the same. This indicates that the ideal curve has been reached, and there is no better possible solution for the given project duration. In Figure 8, lengthening cycles FG, HI, and JK involved progressively less steep decreasing cost slopes. Return cycles GH, IJ, and KL involved progressively steeper increasing cost slopes. The final lengthening cycle retraced along LK, the previous return cycle, and, therefore, signalled that point L was on the optimum solution curve.

In order to perform the Phase III calculations, time-cost data are required for each operation. Because the shortening or lengthening cycles for developing the project time-cost curve generally involve operation time changes over only a portion of their possible range, a continuous time-cost function for each operation is required. This introduces the necessity for an approximation for two reasons. First, it is only practical to require the estimator to give a limited amount of data. Generally, the data would be confined to normal and crash times and costs for each operation. Secondly, the methods developed for solving the problem, whether electronic computer oriented or manual, are limited in their ability to handle more than the simplest type of time-cost variations. The approximation generally adopted is that of a straight line variation of cost between normal and crash times. Performance of an operation in any intermediate length of time is accomplished at a corresponding intermediate cost. This is a convenient and workable assumption but some discussion of its reasonableness is necessary before proceeding to the mechanics of the systems made possible by this approximation.

Figure 9 shows several approximations of operation time-cost relationships. Figure 9A indicates the simplest one that has already been described. This is the one commonly used for computer analysis and will be employed in the noncomputer Phase III method that follows. Generally, it is a sufficiently close approximation to make it consistent with other estimating approximations. Figure 9B represents a more complex operation time-cost variation. Its distinguishing features are that the estimator has furnished more than two sets of time-cost data and that each resulting segment slope, proceeding from right to left, is steeper. This type of curve has the merit of often being a closer approximation to actual conditions. Intuitively, it is recognized that for most situations it becomes more and more expensive to obtain additional time savings. This operation time-cost curve can be used in the Phase III calculations with little additional effort. If a straight line approximation is unacceptable, this extra effort is necessary. It is doubtful that this refinement would frequently be justified for single operations, either from the standpoint of the extra effort in applying CPM mechanics or in furnishing estimating data. However, one special situation does justify its use. If a portion of the project network can be replaced by a single operation without otherwise altering any network relationship, this network portion can be treated as a separate "project." The resulting project time-cost curve could then be used as the operation time-cost curve for the substituted single operation. Such an operation curve would have the form shown by Figure 9B. Another operation time-cost relationship is illustrated by Figure 9C Its distinguishing features are that the estimator has furnished more than two sets of time-cost data and that one or more of the resulting segment slopes, proceeding from right to left, is less steep than its preceding one. Here the segment GH might represent one method of performance subject to variations by, say, overtime work or larger crew sizes. Segment IJ might represent conversion to an entirely different method again subject to variations by overtime work or changes in crew sizes. The segment HI, which might be vertical, would indicate the additional cost of one method over the other. This shape of time-cost curve cannot be handled simply as was possible in the case of the Figure 9B curve. If it can be handled at all by an electronic computer solution, only the largest capacity computers now in existence would probably be able to do so. The value of the additional accuracy compared to the cost of the additional computational effort makes it doubtful if computer programs to handle this type of cost-time curve will become available in the near future. Fortunately, a straight line, such as GJ, eliminating the reverse curve effect, generally offers an approximation that is still reasonably satisfactory. Figure 9D represents another situation that sometimes arises. This is the case of a discontinuous operation time-cost curve where a continuous straight line from K to N would not be a justified approximation. Examples sometimes arise in materials procurement operations. To illustrate, consider Operation No. 3 of the network shown on Figure 2. This operation involved the procuring of steel piling for use as pier fender piles. For the lengths required by this West Coast job, piling could only be purchased on the East Coast. The curve segment KL of Figure 9D might represent several quotations and modes of shipment. Time might vary from 60-75 days. An alternative was to obtain shorter sections locally and butt weld them to required lengths. The costs were considerably higher but the time range was from 10-15 days. Segment MN might represent this possibility. Between 15 days and 60 days there were no intermediate solutions. The problem was solved in the case of the project of Figure 2 by two complete solutions of the project time-cost curve, one based on using a time-cost curve for Operation 3 equivalent to segment KL and one using a curve equivalent to segment MN. The two project time-cost curves were superimposed, graphically, and the most favorable sections of each used for scheduling decisions. Obviously, this procedure is not a very satisfactory one if many such operations are involved. The manual approach would rule it out to a much greater degree than a computer approach. The alternative is to use the continuous straight line approximation for the operation and make adjustments at the end of the problem to eliminate any inconsistencies that result.

The acceptability of the operation time-cost curve approximation is one of three basic assumptions on which the Critical Path Method is founded. These assumptions will be discussed in the section "Evaluation and Applications" near the end of this report.

PHASE III - NORMAL START

The mechanics of a noncomputer solution for CPM Phase III will now be presented using the same example as for Phases I and II. In this section of the report the project normal time-cost point will serve as the starting point. This is the usual case for computer solutions and corresponds to point A of Figure 8. Required data will be the same as that for computer programs and the results will include all information obtainable by computer methods. The method is based on the assumption of linear time-cost variations for each operation as indicated by Figure 9A, and upon the assumption that a realistic project network chart has been developed in Phase I as indicated by Figure 3. A detailed description of the required steps will be given, but the worksheets are, in most cases, in final form as they appear at the end of the procedure. For the reader interested in following the details of the method, it would be advisable to prepare fresh forms and work through the problem as it is discussed. This applies particularly to the worksheets shown on Figure 11, 12, 13, and 16. Figures 14 and 15 have been shown in separate stages as they appear after each necessary revision.

It should be pointed out here that there will be a strong temptation for a busy person to bog down and quit on the first attempt to follow this procedure. It is involved and takes time to grasp. However, it is mechanical and repetitive. Once the analyst understands it, the solution becomes almost automatic.

Figure 10 contains the necessary new data for Phase III calculations. These include the operation costs at normal production rates (normal cost equals least possible cost) and crash times and corresponding crash costs for each operation. Normal times are brought forward from Figure 5A. The differences between normal time and crash time and between crash cost and normal cost for each operation are computed as shown. Then the cost difference is divided by the time difference to obtain the cost per day for shortening each operation. This value represents the slope of line AB in Figure 9A. The two operations which the estimator says cannot be shortened have no slopes. At the extreme right side of Figure 10, but using the estimated crash times, the earliest finish time of the final operation is computed in the same manner as was done using normal times on Figure 5A. This step is not essential, but it does determine the project crash time quickly and provides a check point for the project shortening process. The following data sums up what is known about the project time-cost curve at this stage:

Normal (least) cost = $52,500
Normal time = 64 days
All-crash cost = $62,630
Crash time = 49 days

To obtain data for the project time-cost curve between normal time and crash time limits as well as corresponding operation schedules, requires six worksheets. These are briefly described as follows:

Worksheet No. 1 (Figure 11): This is a summary sheet on which data from other sheets is entered as developed. Computations are performed by cycles, and each cycle produces a new segment of the project time-cost curve. First, the combination of critical operations having the least cost slope is determined and entered. These operations are shown in the columns labeled "Min. Op. Slope Comb." Next, the shortening limitations for these operations are determined and entered. One such limitation is the days of possible shortening for each operation to reach crash time. These limits are shown for their respective operations in the columns labeled "Op. Poss. Shortening." The other shortening limitation is determined by interaction effects in the network that produce new relationships requiring other worksheets to be updated. It is entered in the column labeled "Interact. Limit." The governing limitation is entered in the column "Days Change" and is used with the cost slope to calculate a cost change for the cycle. Finally, new project cost and time values are computed. Then the next cycle is commenced.

Worksheet No. 2 (Figure 12): The left portion of this sheet is a tabulation of project operations arranged in order of increasing cost slopes. Entries indicate whether an operation is critical or not by showing the cycle in which it becomes critical in the column labeled "C.P. Cycle." Entries in the column "Finish Cycle" indicate the cycle in which a critical operation has been shortened either to its crash limit or to a point at which further shortening would make it noncritical (NC). The foregoing information assists in selecting the most favorable operations available for shortening. The right portion of this sheet is a tally record on which entries indicate the revised operation times and the remaining possible shortening as changes take place. The original "Normal Time " and "Poss. Short." are entered from Figure 10. The latest entry for an operation provides one of the shortening limitations for the cycles of Worksheet No. 1. The entries also provide complete operation scheduling information at the end of each shortening cycle.

Worksheet No. 3 (Figure 13): This sheet provides the other shortening limitation for the cycles of Worksheet No. 1. This limitation is determined by changes in the network relationships that require updating of Worksheets Nos. 4 and 5 and possibly Worksheets Nos. 2 and 6. The latter sheets are involved when the updating of Worksheet No. 5 indicates that critical operations have been added or lost.

Worksheet No. 4 (Figure 14): This sheet is known as a precedence matrix. It indicates all of the additional operations that are affected by a change in any specified operation and provides this data for use on Worksheet No. 3. It must be updated at intervals as indicated by information developed on Worksheet No. 3.

Worksheet No. 5 (Figure 15): This sheet is the project network. It is used to determine the conversion of noncritical operations to critical ones, or vice versa. Each connecting line of the project network shows that one operation follows another. The term "lag" has been adopted in this manual method to indicate the days between the earliest completion of an operation and the earliest start of a following operation. These lags for each connecting line are calculated on Worksheet No. 3. When a lag is, or becomes, zero, the corresponding connecting line is doubled and shows that one operation immediately follows another. When visual observation indicates a continuous chain of doubled connecting lines, a critical path has been formed. The corresponding connecting lines are then tripled to show this path. Whenever a critical path is added or lost Worksheets Nos. 2 and 6 must be updated.

Worksheet No. 6 (Figure 16): This sheet provides the combinations of operations having the least cost slope for entry on Worksheet No. 1. It is only required when there is more than one critical path. The critical paths are listed in pairs on the upper portion of the sheet. On the lower portion a systematic trial and error procedure determines the optimum combination of operations.

A more complete description of the use and development of these worksheets will be included as procedural steps are discussed.

The starting point is cycle 0 on Figure 11. The normal project cost and time are entered. Cycle 1, the first shortening cycle, is next. The first question is "Which operation, or operations, should be shortened to produce project shortening at the least increase in project cost?" Three factors determine the choice. The operation must be critical; it must be possible to shorten it within its crash limit; and, since there is only one critical path at this time, it must be a single operation that has the least possible cost-time slope. A selection might be made from Figure 10, but it is much easier to first rearrange the operations in increasing order of cost slopes as shown on Figure 12. The third column on Figure 12 shows the shortening cycle at the end of which the operation became critical. At the outset of shortening the only entries in this column are the zeros opposite Operations Nos. 3, 13, 15, 17, and 18. These are the critical operations as determined in Phase II on

Figure 5C and as also shown on the project network of Figure 15A. The normal time, possible shortening, and revisions of these figures occupy most of the remainder of Figure 12. The only remaining entries are in the fourth column, and they indicate the cycle at the end of which the operation can, or should, no longer be shortened. One basis for such an entry is that the remaining possible shortening has reached zero as indicated on the tally portion of the sheet. The other basis is that any further shortening will make the operation noncritical. This is determined by data on Figure 16 which will be discussed later. Examples of such entries are those followed by the letters NC. Having developed Figure 12, it is simple to select the operation meeting the three criteria given. It will be the first operation, starting at the top of the tabulation, that has an entry in column 3 and no entry in column 4. For Cycle 1, Operation 3 satisfies this requirement. (At this time there are no entries in column 3 opposite Operation 5 or in column 4 opposite Operation 3.)

Having selected the operation to be shortened, the next question is how much can it be shortened? There are two limitations that determine the answer. One is the amount of possible shortening of the operation, which can be obtained from the latest possible shortening time entry on Figure 12. For Operation 3 at the beginning of Cycle 1 this is 12 days. The second limitation is referred to as the interaction limit. This limit represents a point at which network relationships change and, possibly, a new critical path is introduced. Determination of this limit is the most involved problem of the Phase III procedure. Several methods have been explored and the one offered here appears to be the most workable.

Figure 13 provides a worksheet for interaction limit determination. It is used in conjunction with a triangular matrix, known as a "Precedence Matrix," and the project network chart. The first two columns of Figure 13 give the operation numbers that determine, respectively, the beginning and end of every connecting line in the project network. Starting with Operation 1 and referring to the network chart of Figure 3, there are two connecting lines originating at this operation, 1 to 5 and 1 to 6. Operation 1 is listed as the preceding operation, or

Pre. Op., in column 1 and Operations 5 and 6 are listed as the following operations, or "Post Op.", in column 2. This process is continued until every operation appears in column 1, in numerical order, and every connecting line in the network is represented by a column 1 - column 2 pair of numbers. Next, in column 3, the earliest finish time, from Figure 5A is entered for the operation appearing in column 1. In column 4, the earliest start time, from Figure 5A, is entered for the operation appearing in column 2. The differences between the earliest start times of the following operations and the earliest finish times of the preceding operation are entered in column 5 under the heading of "Lag." Our results show, for example, that between Operations 1 and 5 there is zero lag. This indicates that when each operation is performed at its earliest possible time, Operation 5 commences immediately upon the completion of Operation 1. Between Operations 4 and 14, however, there is a lag of five days, indicating that Operation 14 does not start until five days after the completion of Operation 4. The word "lag" has been introduced here instead of the word "slack", since "slack" has become generally used in the probabilistic scheduling cases as the counterpart of the word "float" in the deterministic cases.

From the data developed so far on Figure 13, the precedence matrix can be constructed. On a square grid system, shown on Figure 14A, the operation numbers for all operations are listed vertically along the left border and horizontally across the top. The numbers on the left border represent preceding operations for network connecting lines, while those along the top represent following operations for network connecting lines. The zero lag relationships developed on Figure 13 are entered on this matrix by entering zeros in the proper grid boxes which represent the preceding operation and following operation of all connecting lines having zero lag. No entries are made where Figure 13 indicates a positive lag. For example, Figure 13 indicates that Operations 8, 9, and 10 each follow Operation 6 with lags, respectively, of 0, 0, and 4 days. This information is transferred to the precedence matrix by entering on the row of Preceding Operation 6 a zero in the column of Post Operation 8 and a zero in the column of Post Operation 9. No entry is made in the column of Post Operation 10. To complete the first stage of the precedence matrix, shown as Figure 14A, one entry is made for every connecting line in the network that has zero lag as tabulated on Figure 13. A diagonal line bisecting the square grid is drawn for use in the following stages of precedence matrix development and updating. The information shown separately on Figures 14B to 14G would normally be developed on the same original sheet as Figure 14A The procedure is shown in separate stages in this report merely to facilitate the explanation.

On Figure 14A the precedence matrix indicates only those operations which immediately follow other operations. On Figure 14B the precedence matrix is developed further to indicate, by the addition of x entries, the entire chain of operations that immediately follow one another, timewise, starting with the operation of the row in which the entries are made. For example, Figure 14B indicates that any time variation in Operation 13 would affect Operations 15, 17, and 18. For a graphical illustration, visualize a bar chart representing the operations of this project. If Operation 13 were shortened by one day, the bar representing Operation 13 would be shortened by one day, and the entire bars representing Operations 15, 17, and 18 would be shifted one day to the left. (An exception to this statement will be explained below.)

The x entries on the precedence matrix are obtained by starting in the last row of the grid which contains a zero. Commencing at the intersection of the diagonal line with that row, move vertically upward in the column containing this intersection to squares containing zero entries. When zero entries are encountered, place an x in their rows directly above the zero entries (and in subsequent steps also above the x entries) of the starting row. To illustrate this procedure with reference to Figure 14B, the last row containing a zero is that of Operation 17. This zero states that Operation 18 immediately follows Operation 17. Starting where the diagonal line intersects this row, proceed up column 17 to the only zero encountered, which is in the row of Operation 15. This zero states that Operation 17 immediately follows Operation 15. An x is entered in this row in column 18, directly above the zero in row 17. This x states that Operation 18 is directly affected by any change in Operation 15. The reason is that Operation 18 immediately follows Operation 17 which immediately follows Operation 15. Having completed the entries originating from the bottom row, return to the next row up containing a zero and repeat the same procedure. This would be row 15. Proceeding up column 15, the only zero encountered is in row 13. X's are entered in row 13 directly above the zero and x entries in row 15. (The entries in row 13 now give the information discussed in the example of the preceding paragraph.) This procedure is repeated a row at a time until the top of the grid is reached.

There must be at least one zero in every column of the precedence matrix (except those representing initial operations), because every operation can immediately follow at least one other operation. There may be more than one zero in a column. This raises a detailed point that it is necessary to discuss. Suppose there were another zero in column 15 in row 12 (a possibility since connecting line 12-15 has a positive lag subject to change as shortening takes place). It was previously stated that shortening Operation 13 by one day would cause the bar representing Operation 15 on a bar chart to shift left by one day. This is another way of saying that both the earliest start time and earliest finish time of Operation 15 would be reduced by one day. However, if Operation 15 also directly follows Operation 12, its bar could not shift unless Operation 12 either was shortened or shifted also. The mechanics of the system automatically take care of this problem. There are two possibilities. If Operation 15 is critical, then both Operations 12 and 13 would be critical since they immediately precede Operation 15. Therefore, Operation 13 would not be shortened unless either Operation 12 or some operation in its critical chain were shortened concurrently. Therefore, Operation 15 could shift as stated. However, if Operation 15 were not critical, Operation 12 would not necessarily be critical. If it isn't, shortening Operation 13 would not require an accompanying change in Operation 12. Therefore, if the finish time of Operation 12 did not change, the start and finish times of Operation 15 could not change. Rather, connecting line 13-15 would change from a zero lag to a positive lag. This would be indicated on

Figure 13 for the cycle involved, and the precedence matrix would have to be updated. This updating would require removing the zero in the grid box for connecting line 13-15 and also removing all x's from the matrix that have been entered solely as a result of that zero. A more accurate statement than that made earlier concerning the relations shown by the precedence matrix can now be offered. The zero and x entries in any row of the precedence matrix indicate the following operations for which scheduling times may be affected by a shortening of the operation given by the row number. Whether these operations are actually affected is determined by one cycle of entries on the Network Interaction Limit (Figure 13) worksheet. If a zero lag becomes a positive lag, one or more entries on the precedence matrix will be removed. The scheduling times for the corresponding operations will not have been affected by the operation shortened.

Note that the preceding statement was confined to operation shortening. If operation lengthening is involved the corresponding statement becomes simply: The zero and x entries in any row of the precedence matrix indicate the following operations for which scheduling times will be affected by a lengthening of the operation given by the row number. Generally, contract change orders and extra work orders involve lengthening of operations. It is convenient to refer to a row of the precedence matrix and state definitely which operations will be affected. It is still advisable to work through Network Interaction Limit cycles for the lengthening time required, because some positive lags may become zero causing additional entries on the precedence matrix.

Having developed the initial precedence matrix on Figure 14B, the interaction limit for the first cycle may be determined on Figure 13. Under the cycle 1 heading and on the rows opposite the appropriate "Pre. Ops.", entries are made in the left half of the column to indicate which Pre. Ops. have their earliest finish date shifted to an earlier time by the shortening of Operation 3. There are two bases for these entries. One is that the operation being shortened will have its earliest finish date shifted. The second is that all operations with a zero or x on the precedence matrix in the row of the shortened operation will have their earliest finish dates shifted. These two criteria indicate that entries should be made opposite Pre. Ops. 3, 13, 15, and 17. (There is no Pre-Op. 18.) Next, a similar entry is made in the right half of the column to indicate which Post Ops. have their earliest start dates shifted. There is only one basis for these entries. It is the information from the precedence matrix indicating, again, the same operations affected by a change in the shortened operation. These are Operations 13, 15, 17, and 18. These entries are less convenient to make since the Post Ops. are not listed in numerical order. For each operation that is affected as shown by the precedence matrix, a run must be made down the Post Op. column. A valuable check on the work can be maintained if the network chart is referred to when making the entries for each operation. The number of connecting lines terminating in an operation on the chart indicates the number of entries that should be made for that operation in the right half of the column. For example, the chart of Figure 3 (or 15A) indicates that three connecting lines terminate at Operation 17. Therefore, there are three entries to be made. A careful run down the Post Op. column will show that 17 appears three times. Such a check will eliminate the possibility of omitting entries.

The horizontal V symbols in the cycle 1 column of Figure 13 indicate all shifts in earliest start and finish dates. They are pointed to the left in keeping with the bar chart concept of changes to earlier dates. If there are no entries in a row this indicates that neither the finish of the preceding operation nor the start of the following operation will change, and, therefore, the lag will not change. Two horizontal V's in a row indicates that both operations will shift, and there will still be no change in the lag. A single horizontal V in the left half of the column, for a shortening cycle, indicates that the earliest finish of the preceding operation will shift to an earlier date, and the lag will be increased. The fourth, and only other possibility, is a single horizontal V in the right half of the column which indicates that the earliest start of the following operation will shift to an earlier date, and the lag will decrease. When a lag decreases from any positive value to zero, the network relationships are altered and the precedence matrix will change. At this time an additional critical path may or may not be introduced. In the example used here, a new critical path is added each time a lag becomes zero. This does not have to be the case. But, in any event, when a lag becomes zero, changes take place that require an interruption to update the data. By running down the cycle 1 column and observing the lags for those connecting lines for which the entries indicate that lags will be shortened, the least of these lags is determined. This is the "Interaction Limit." In the example, it is equal to 2 days and is established by the lag of connecting line 11-17.

After the interaction limit has been determined for cycle 1, it is entered on the Summary Sheet, Figure 11. Entries for cycle 1 may now be completed on this sheet. The number of days of shortening for a cycle is determined either by the interaction limit or by the possible shortening of the operations involved. The smallest number must be used In this case, cycle 1 produces 2 days of shortening as controlled by the interaction limit. Since Operation 3 has a cost slope of $50 per day, project costs are raised by $100 and project time is reduced to 62 days.

Before proceeding with cycle 2 the supporting data sheets must be updated. The actual number of days of shortening is entered on Figure 13, and all of the lags for which cycle 1 entries indicated changes are revised by this number. Only those that are changed need be entered in the revision column. Whenever a positive lag becomes zero or a zero lag becomes positive, a revision to the precedence matrix must be made. In this cycle, the lag between Operations 11 and 17 was reduced from 2 to zero. The revised precedence matrix is shown as Figure 14C. Revisions are indicated by the bordered blocks. One comment is warranted. When a zero is entered in the square of row 11 - column 17, it is necessary to backtrack downward before the new data can be carried upward on the matrix. To illustrate, starting at the newly entered zero, drop down column 17 to the row of the diagonal line intersection. Then carry up to row 11, as x's, any zero or x entries in this row. In this case there is a single zero in the row of the diagonal intersection, row 17. An x must be placed in this column in row 11. Now proceed to carry the new entries of row 11 up the chart in the usual manner. The precedence matrix is now ready for cycle 2.

It was mentioned that when a lag becomes zero a new critical path may or may not be added. It is essential to determine whether or not this has happened and, if so, to identify it. This can most easily be done by visual means using the network chart. As in the case of the precedence matrix, a single network chart would be used and updated as necessary. To clarify this explanation, successive revisions to the network chart are shown as Figures 15A to 15F. On the initial network chart of Figure 15A, all connecting lines representing zero lag have been drawn double. Where these doubled connecting lines form a chain from start to finish of the project, they represent a critical path. In this case they have been shown as triple lines to further distinguish them. Other means, such as colors, might serve to classify their status. On Figure 15A, the connecting lines between Operation 3, 13, 15, 17, and 18 have been tripled to represent the original critical path through the network. Additional critical paths need not form a complete new chain from end to end of the project. They may only involve new subchains between either end of the project and some operation on an existing critical path, or between intermediate points on existing critical paths.

At the end of cycle 1, the network chart would be updated by doubling connecting line 11-17, the lag of which has become zero. This is shown on Figure 15B. A visual observation indicates a new critical subchain between the beginning of the project and Operation 17. This subchain consists of Operations 1, 5, 7, 11, and 17. The corresponding connecting lines are tripled to indicate the second critical path.

Figure 12 must also be updated before commencing cycle 2. A revised possible shortening of 10 days is entered in the cycle 1 column opposite Operation 3. While it is not essential to enter this information in a separate column for each cycle, it adds convenience if the analyst wishes at the completion of the problem to quickly pick off the entire operations schedule as of the end of any particular cycle. It also facilitates checking the work. Since a new critical path has been developed, this is indicated by entries in the third column opposite the new critical operations. The "1" signifies that the operation became critical as a result of the changes made during cycle 1.

The fact that there is now more than one critical path introduces an additional problem in selecting the operation, or operations, to shorten for the next cycle. Where the critical paths are parallel, as they are between the beginning of the project and Operation 17, shortening a critical operation on one path necessitates shortening a critical operation on the other path; otherwise the entire project would not be shortened. The alternative is to shorten a single critical operation common to both paths, such as Operations 17 and 18. The problem is to determine the least costly choice, whether it is to shorten a combination of two operations simultaneously or to shorten a single operation. While this is not particularly difficult to do if there are only two critical paths, the decision rapidly becomes more complex as additional critical paths are added. Therefore a system must be established for making the decision. Figure 16, used in conjunction with Figure 12, presents such a system. For explanation purposes, Figure 16A presents the initial version of this worksheet and Figure 16B presents the final version after the last cycle of shortening is completed.

At the top of Figure 16A under the heading of Cycle 1, are shown the two subchains, 3-13-15 and 1-5-7-11, that, as a result of cycle 1 shortening, have become simultaneously critical between the beginning of the project and Operation 17. The operation in each path that has the least cost slope is circled for quick reference. The operations in each path are listed vertically according to their sequence steps. At the left of the tabulation all possible operations in each sequence step are indicated as an index. This index is not essential, but as more critical paths are introduced or as projects with larger numbers of operations are analyzed, this procedure makes it easier to locate the paths containing a given operation. For example, if Operation 12 appeared in any critical path, it would be in the row containing the 11 to 14 sequence of operations. For only two paths and a few operations it is just as easy to scan the two columns for Operation 12.

On the bottom portion of Figure 16A all the operations subject to change are listed, as on Figure 12, in order of increasing cost slopes. The cost slopes are also relisted for convenience. The process of selecting the most favorable combination of operations to be used for cycle 2 shortening is now commenced. The most eligible single operation, according to our previously developed criteria for using Figure 12, is Operation 5. An x is placed opposite Operation 5 on the bottom portion of Figure 16A in the first column under the heading of cycle 2. A check of the top portion of Figure 16A indicates that Operation 5 does appear in one of the portions of a critical path that has a parallel critical path. Therefore, nothing is gained by shortening Operation 5 alone. In addition, the least expensive operation in the parallel path must also be shortened. The circled number indicates that this is Operation 3, and an x is placed opposite it in the lower table. The sum of the cost slopes of shortening these two operations, $90 per day, is entered at the foot of the column. This is not necessarily the best solution as there are other operations to try that may give a smaller figure. The next eligible operation according to Figure 12 is Operation 3. This is one of the operations of the first combination and need not be tried again, since the same results would be obtained. The next eligible operation is Operation 15, which is entered on Figure 16A in a new column under the cycle 2 heading. By repeating the previous procedure it is determined that Operation 5 would have to be concurrently shortened. An entry is made accordingly, but no attempt is made to sum the cost slopes. The fact that one of the two operations is the same as that entered in the first column under the cycle 2 heading, while the second operation falls further down the listing, indicates that a higher sum would result. There is still no basis for discontinuing further trials. Figure 12 indicates Operation 17 as the next eligible candidate. After the entry is made on Figure 16A, a check of the upper portion of the worksheet indicates that there is no parallel critical path to that containing Operation 17. (Operation 17 is common to both critical paths.) Therefore, the entire project can be shortened by shortening Operation 17 alone at a cost slope of $70 per day. There is no need to proceed further, since Figure 12 indicates that the next eligible operation for trial is Operation 13 at a cost slope of $180 per day. Operation 17 is entered on the Summary Sheet, Figure 11, as the minimum slope selection for cycle 2. To determine how much Operation 17 may be shortened involved the entire procedure developed for cycle 1. There are no new concepts required for this cycle.

All of the steps and worksheets for solving the remainder of the problem have been developed at this point. A third, fourth, fifth, and sixth critical subpath is introduced, respectively, by cycles 2, 6, 7, and 8. The least cost combination of operations to be concurrently shortened requires a maximum of four operations in cycle 7. The limit of project shortening has been reached when there are no critical operations that may be shortened effectively as shown by column 4 of Figure 12. In the example, entries in this column indicate that every critical operation has reached its shortening limit after nine cycles. This limit should check the predetermined crash limit calculated at the outset of the Phase III work.

In column 4 of Figure 12, several of the entries are for operations which have not been shortened to their crash limits. These entries give a cycle number followed by the letters "NC". This means that as a result of the changes produced in the given cycle, any further shortening of the corresponding operation would cause it to be noncritical. The basis for such an entry comes from the critical path comparisons in the top portion of Figure 16B. When it becomes impossible to shorten one of two parallel critical subpaths because all operations in the path have been shortened to their crash limit, then the effect of shortening an operation in the other parallel path is to make that path noncritical. The operations in the path would, therefore, become noncritical unless they are also in other independent critical paths. These possibilities are illustrated by the example. At the end of cycle 8, subchains 2-10 and 3 become parallel critical subpaths between the beginning of the project and Operation 13. However, Operation 3 had, in that same cycle, been shortened to its crash limit. Any shortening of Operation 2 (Operation

10 had also previously been shortened to its crash limit) would cause the subchain 2-10 to be noncritical between the project beginning and Operation 13. However, it does not follow in this case that shortening of Operation 2 would cause it to be noncritical since it is part of another critical path that is independent of Operation 13. Therefore, no entry was made in column 4 of Figure 12 at the end of cycle 8, and Operation 2 was still considered in arriving at the least slope combination for cycle 9. Had Operation 2 been one of those shortened in cycle 9, the lag of connecting line 10-13 would have increased from zero to a positive value, and that connecting line would no longer have been on a critical path on the project network chart.

At the end of cycle 9, all operations in the critical subpath 1-5-7-11 have been shortened to their crash limits as indicated by the fact that these numbers have been struck out on the tabulation under cycle 1 at the top of Figure 16B. It follows that shortening either Operation 13 or 15 would make the parallel critical subpath, 3-13-15, noncritical between the project beginning and Operation 17. Since this is the only critical path containing Operations 13 and 15, it can be said that they would become noncritical operations if shortened further. This justifies the entries for these operations in column 4 of Figure 12. Moving to the next pair of critical paths on Figure 16B, a similar justification may be made for Operations 2 and 14. While Operation 2 is common to two independent critical paths, both of these paths would now become noncritical if Operation 2 were shortened. In a similar fashion, the remaining pairs of critical subpaths give justification for the entries in column 4 of Figure 12 for Operations 4 and 6. If, by oversight, an operation that would become noncritical upon shortening were used for a shortening cycle, an interaction limit of zero would result on the Figure 13 worksheet. This would demand an investigation which should point out the oversight.

To sum up the mechanics of performing the Phase III calculations, a list of ten procedural steps for each cycle follows:

1. Determine the critical operation, or combination of critical operations with least cost slope. Use Figure

12. If there is more than one critical path, also use Figure 16.

2. Enter the selected operations opposite the shortening cycle number on Figure 11. Also enter the possible shortening for each operation, and the cost slope of the combination. Data for this comes from Figures 12 and 16.
3. Enter operations to be shortened on Figure 13 and proceed to determine the interaction limit with the aid of the precedence matrix, Figure 14.
4. Enter the resulting interaction limit on Figure 11, determine the days the project is to be shortened, and complete the cycle data, thus obtaining a new Total Project Cost and Total Project Days.
5. Enter the days shortened on Figure 13 and revise all lags that change during the cycle.
6. If any positive lag has become zero or any zero lag has become positive, update the precedence matrix, Figure 14.
7. If any positive lag has become zero or any zero lag has become positive, update the project network chart, Figure 15.
8. If the project network chart indicates a new critical path (or loss of a critical path), update column 3 of Figure 12 and the upper portion of Figure 16.
9. Update the tally portion of Figure 12. If any operation has reached its crash limit, update column 4 of Figure 12.
10. If any entries have been made in column 4 of Figure 12, check the top portion of Figure 16 and strike out the corresponding operations. If a circled operation is struck out, circle another operation in the same column with the next least cost slope. If all operations in a column have been struck out, check to see whether further entries for noncritical shortening limitations

should be made in column 4 of Figure 12. If such entries are made, repeat this step.

Figure 17 illustrates graphically the information from the Summary Sheet, Figure 11, for the entire project shortening curve. From this graph, for any given completion time, the least total project direct cost can be determined. Moreover, the scheduling for every operation necessary to meet this project time and cost may also be determined. The scheduling is obtained by deducting from the normal time of each operation the number of days of shortening for those operations indicated by the graph between the project normal time and the specified time. A more direct approach would be to refer to the tally portion of Figure 12. The complete schedule at the end of the cycle in which the specified project completion time falls is available. Those operations shortened during that cycle would be increased by the number of days between the specified project time and project time at the end of the cycle.

If the procedure of Phase III is carried from the project normal time to the project crash time, the full range of optimum schedules and project costs for any possible completion time is available. Remembering that only direct costs have been considered, one use of the project time-cost curve is to combine it with other time-cost data relating to project performance. From a contractor's viewpoint, a second curve reppesenting indirect costs could be developed. In those cases where there were bonus-penalty provisions, a third curve could be added representing this incentive for completion earlier than the specified time. All curves would be plotted on a common basis of either working days or calendar days. A summation of the curves would result in a project time-total cost curve having a minumum point. The schedule corresponding to this minimum point would be the optimum schedule since it would give the lowest total project cost.

It is not essential in many cases that the entire project time-cost curve be developed. Initially, the contractor might carry the calculations only far enough to obtain project compression to the point of specified completion time. This would give him the best schedule to meet the project duration requirements of the owner. A further refinement from that point might then be made by considering the indirect cost curve, as described above. It still might not be necessary to develop the entire project time-cost curve. A slight additional projection of it, added to a corresponding segment of the indirect cost curve would usually indicate the minimum point of the combined curves. The entire indirect costs need not be developed to plot the indirect cost curve. Only the variable portion of the indirect costs over the time range between project duration limits is necessary for obtaining the minimum point of the combined curves.

PHASE III - ALL-CRASH START

As demonstrated, the normal time-least cost point on the project time-cost curve is one possible starting point for Phase III scheduling variation. The other known point on the project time-cost curve at the beginning of Phase III calculation is the crash time-all crash cost point. This may also be used as a starting point. Referring to Figure 8, this would mean starting at Point B rather than Point A.

One practical reason for starting at the all-crash point is that it may be closer to the final scheduling solution than the normal point. This could be the case where an owner agency has set a completion date that only a nearly crash effort will satisfy. It is particularly advantageous to start at the closest point to the final solution when calculations will only be made to the point satisfying the specified time schedule. On the other hand, if the entire curve AC of Figure 8 is to be developed, it is more advantageous to start at normal point A and thereby eliminate the necessity for developing the curve portion BC.

A second and more important reason for developing a solution commencing at the all-crash point is to present the mechanics for moving in the opposite direction along the project time-cost curve. This procedure involves changes in noncritical as well as critical operations. A detailed discussion of the entire solution is not necessary since it is similar to that developed in the last section. Only those points that are different or troublesome will be considered here.

The initial input data required from the estimator is the same as that needed for Phase III - Normal Start. Therefore, Figure 10 may again serve as the source of input data. The Summary Sheet, Figure 18, has the same form as Figure 11. Two headings are varied slightly to indicate that the maximum slope combination is to be used in place of the minimum slope combination and that cycles may be limited by possible lengthening of operations rather than possible shortening of operations. On Figure 18, the eight initial cycles of Phase III involve lengthening noncritical operations. This changes total project costs but does not change project time. In addition there are the nine cycles previously found necessary to develop the project time-cost curve when working in the opposite direction.

The Operation Selection Sheet, Figure 19, has the same general form as Figure 12. The principal difference is that the operations are arranged in order of decreasing cost slopes instead of increasing order. Many of the numbers in column 3 are struck out during the lengthening process. During the lengthening of critical operations in the later cycles, most of the critical paths tend to become noncritical, which explains why the numbers are struck out. On Figure 19 there are no "NC" entries in column 4 as there were on Figure 12. A little thought will indicate that these entries are not necessary in the lengthening process.

Figures 20A and B, the Interaction Limit Determination Sheets, are the same as Figure 13 except that the word "lengthened" is substituted for "shortened". The horizontal V symbol for indicating a change in either an earliest finish date or an earliest start date is pointed toward the right instead of the left. This is consistent with the visualization of changes taking place in a corresponding schedule bar chart. As operations are lengthened, the necessary shifts occur toward the right, or in the direction of longer time. As before, the absence of entries in a column or entries in both halves of the column indicate no change in the lag. A single entry in the right half of the column indicates that the earliest starting date of the following operation has shifted to a later date, and the lag is increased. A single entry in the left half of the column indicates that the earliest finish date of the preceding operation has shifted to a later date, and the lag is decreased.

The Precedence Matrix, Figure 21, is developed and used in the same way as Figure 14. Only the initial form of the precedence matrix is given. The final form at the end of cycle 17 will be the same as the initial form for the shortening solution. Figure 14B. One part of the procedure of updating the precedence matrix deserves comment. In the lengthening process it is more common for zero lags to change to positive lags. This generally requires the removal of x entries as well as that of the zero involved. Extreme care must be used to delete only the x's that were introduced solely by this zero. The first step is to remove the appropriate x's in the row of the deleted zero. This requires backtracking downward one step to determine which x's were brought up by this zero. If there are remaining zeros in the row of the deleted zero, then a further check must be made to be sure that any x's being removed were not also brought up as a result of these other zeros. If they were, they should not be removed. Next, the x's in the related rows above that of the deleted zero are removed in a reversal of the process by which they were entered. Again, if there are zeros in such rows besides the one related to the deleted zero, checks must be made to be sure that none of the x's being removed are also present because of these zeros. If they are, they should not be removed.

Figure 22 gives the initial form of the project network chart. The final form would be the same as Figure 15A, the initial form for the shortening process. The lengthening process requires frequent removal, as well as addition, of lines in this chart. When a zero lag becomes positive, the appropriate line is removed from the chart. This may indicate that a certain path ceases to be critical. If so, the lines that represent the critical condition for this path must be removed. Moreover, the corresponding entries in column 3 of Figure 19 must be struck out, and the appropriate pair of parallel critical paths on the upper portion of Figure 23 must also be struck out.

Figure 23 is the final worksheet for the lengthening procedure and is very similar to Figure 16B for the shortening procedure. Noncritical operations to be lengthened in the initial cycles are selected one at a time. Therefore, this selection does not require the use of Figure 23. In the first eight cycles, the operations chosen were picked from those available in order of descending cost slopes. This is an arbitrary basis for the selection of noncritical operations, and does not necessarily have to be followed. However, when critical operations are lengthened, the procedure demands that a combination of operations producing the greatest decrease in project cost per day of project lengthening be chosen. To determine such a combination, the highest available critical operation from Figure 19 is selected as a first trial and an x entered in the appropriate space in the lower portion of Figure 23. By examining the parallel pair of critical paths in the upper portion of the figure, the additional critical operations that may be simultaneously lengthened are obtained. From each parallel critical path the operation with the highest cost slope, that still can be lengthened further, is selected and additional entries are made in the column of the first x entry on the lower portion of Figure 23. On this top portion of the figure, the circled operations are those having the highest cost slopes rather than the lowest as on Figure 16B. When operations on the upper portion have been lengthened to their limit they are struck out. The fact that all operations in one path have been struck out does not prohibit operations in the parallel path from being lengthened. When a path becomes noncritical, the pair of paths of which it is one is struck out.

Having obtained the maximum possible number of operations, selected as above, in a column on the lower portion of Figure 23, their combined cost slope is totaled. Additional trials usually are necessary, each starting with the next lower available and untried operation from Figure 19. Trials cease when it becomes certain that no combination of remaining operations can produce a higher cost slope than already obtained. One potential source of error should be pointed out. No operation may be used in a combination when that operation appears in a critical path that already contains one of the other operations of the combination. For example, in the first trial of cycle 10 on the lower portion of Figure 23, the combination of Operations 3, 11, and 14 gave the maximum possible combined slope. Since Operation 3 is in a parallel critical subpath with Operations 2 and 10, as entered in the top portion of Figure 23 at the end of cycle 9, it would appear possible to also lengthen one of these operations, say Operation 2, simultaneously. This, in turn, would lead to the lengthening of additional operations, such as Operations 4 and 6. However, both Operations 2 and 10 appear in a critical path including Operation 14. This path has already been lengthened as a result of lengthening Operation 14. Lengthening Operation 2 or Operation 10 would not be the lengthening of a parallel operation but, rather, of a series operation. This is not consistent with the procedure.

Therefore these additional operations must be ruled out from the combination already selected. This possible source of error may be avoided by the following procedural rule: When selecting parallel critical path operations that may be simultaneously lengthened, check to be sure that they do not appear in paths containing operations already included in the combination. If they do, they may not be used.

One further comment on the Phase III All-Crash Start solution should be made. The results shown on the Summary Sheet, Figure 18, are inconsistent at one point with the results obtained by the Phase III - Normal Start solution. Figure 18 indicates that the lowest project cost for the project crash time of 49 days is $58,190. Figure 11 indicates that the least cost for 49 days is $58,130. The latter figure is the correct figure. The inconsistency arises from the fact that there are alternative patterns for accomplishing the lengthening of noncritical operations. The one obtained by the arbitrary rule of choosing operations in order of decreasing cost slopes is not necessarily the best. It may result in network relationships that permit certain noncritical operations to be lengthened more and others less than an alternative pattern of selection. After the first cycle of the lengthening of critical operations, a shortening cycle might be introduced. If the best possible pattern of lengthening noncritical operations had not been used, then this shortening cycle would result in a better network balance that would give a decreased project cost at crash time. One or more pairs of a critical operation lengthening cycle and a critical operation shortening cycle might be necessary to establish the minimum crash cost, if it were desired. For example, at the end of cycle 9 of Phase III lengthening, if a cycle of critical operation shortening were introduced, it would be found that Operations 11, 14, and 15 would be shortened one day at a project cost increase of $820. This would bring the time and cost respectively to 49 days and $58,130. If critical operation lengthening were then resumed, the exact nine cycles obtained in the Phase III - Normal Start solution would be retraced in reverse. The procedure of alternately lengthening and shortening critical operation combinations to obtain a more favorable network balance for a given project duration is the basis for the method of solution to be discussed in the next section.

PHASE III - CONVENTIONAL ESTIMATE START-MECHANICS

At the beginning of Phase III calculations there are two determinable points on the project time-cost curve. So far, procedures have been outlined to indicate how Phase III can be solved by starting at either of these two points. A third procedure that starts at a random point not on the project time-cost curve will now be discussed and its mechanics considered. Further attention will be given to its applications in the next section.

Referring to Figure 8, it was previously pointed out that within the area enclosed by curves ACBDA, there were an infinite number of possible scheduling solutions for any project. Scheduling solutions that meet a specified project duration time exactly are confined to points on a vertical line through the area. Point E of Figure 8 is one scheduling solution for a project time of 300 days. It is possible to move from point E to a point on curve AC which gives the least cost scheduling solution for a 300 day project duration and to recognize when this point has been reached. Further movement in either time direction along the curve can then be made by procedures already discussed.

Figure 24 gives an example of the data that would be required from the estimator, plus some preliminary calculations. It is assumed here that the estimator, using conventional methods, has arrived at a cost estimate and accompanying schedule for the performance of all operations. They give a direct cost of $57,350 for project completion in a specified time of 56 days. The operational times and costs that correspond to this estimate are given in columns 2 and 3. These times and costs were selected to be consistent with the data previously used in Phase III discussions. In other words, the time and costs fall on the straight lines already used to represent operation cost variations between normal time and crash time limits.

The estimator also has been asked at the outset of Phase III calculations to consider each operation in the project and to develop two additional sets of data. One set gives the shortest possible times for the performance of all operations and the corresponding operation direct costs. The second set gives the least costs at which operations can be performed and the corresponding operation times. It is assumed in the example that the estimator arrives at the same data for "crash" performance and "normal" performance as was previously used.

There are now three sets of estimated data representing, respectively, a feasible solution for performance in a specified time, a crash solution, and a least cost solution. The feasible solution probably does not lie on the ideal project time-cost curve but, rather is equivalent to point E of Figure 8. Its distance from the ideal curve will depend on the skill of the estimator, as well as upon the validity of the assumptions on which the ideal curve is based. From these three sets of data are estimated cost slopes for both lengthening and shortening each operation. Earliest start and finish times for each operation have been calculated to provide information for later computation of lags. The earliest finish date of the final operation confirms the 56-day project duration.

Figure 25, the Summary Sheet, is similar to those previously used. One column has been added to indicate, by initials, whether a cycle involves lengthening or shortening an operation and whether it involves critical or noncritical operations. The procedure is to first lengthen all noncritical operations. This results in the equivalent of the vertical line EF on Figure 8. The project cost is reduced without changing the project duration. When the possibilities for lengthening noncritical operations are exhausted, the next step is to lengthen that combination of critical operations that produces the largest cost slope. This is illustrated by cycle 8 of Figure 25; the graphical counterpart on Figure 8 is line FG. Next, the project duration is brought back to the specified duration by shortening combinations of critical operations that produce the least cost slope. This is illustrated by cycle 9 of Figure 25, and the graphical counterpart on Figure 8 is line GH. This procedure of "wiggling-in" is followed until the slope of a lengthening cycle is the same as the slope of a preceding shortening cycle, or vice versa. The point at which the slopes become the same marks the arrival on the ideal curve.

In the example on Figure 25, all lengthening and shortening cycles for critical operations were for one day at a time only. This was dictated by the limits of operational changes for this specific problem, but need not be the case generally. On a critical lengthening cycle, the project duration is changed by whatever limit is established by the shorter of the operation lengthening limits or the interaction limit. On the shortening cycle, the project duration is changed by the amount necessary to bring it back to the specified duration. This might require more than one consecutive shortening cycle, if one cycle failed to bring project length back to the specified duration. While it is suggested here and illustrated on Figure 8 that the shortening cycles serve merely to bring the project duration time back to that specified, it would also be perfectly permissible to carry a shortening cycle to its limit at a less than specified project duration. In this case it may take more than one consecutive lengthening cycle to return to the specified time.

The Operation Selection Sheet. Figure 26, is slightly different than those previously employed since it combines the functions of the corresponding sheets from both shortening and lengthening solutions. The tally portion of the sheet maintains a record of both the remaining possible shortening and remaining possible lengthening for each operation. Column 4 records separately when an operation has reached its limit of being shortened and of being lengthened. Since the operations are arranged in order of increasing cost slopes, selections will be made starting at the bottom of the list on lengthening cycles.

The Interaction Limit Determination Sheet, Figure 27, is very similar to the sheets previously used. One additional row of headings has been added to indicate whether the operations changed during a cycle are being shortened or lengthened. Figure 28 presents the initial precedence matrix. The final precedence matrix would be the same as that following cycle 5 of the shortening solution which is given on Figure 14D. Figure 29 presents the initial project network relationships. The final network relationships are the same as those given by Figure 15C.

Figure 30 is used to determine the appropriate combinations of critical operations for both lengthening and shortening cycles. It is a combination of the sheets used for the two previous solutions. The pairs of parallel critical paths are listed at both top and bottom of the worksheet. Those at the top are used for lengthening cycles. The operations having the maximum cost slope in each column are circled. Operations are struck out when they have been lengthened to their limit. At the bottom of the sheet, circled operations represent those having the least cost slopes in each column. Operations are struck out when they have been shortened to their limits. The middle portion of the sheet contains the trial runs for determining the optimum combination of operations. The procedure is the same as previously explained, except that the bottom portion of the sheet rather than the top is referred to during shortening cycles.

PHASE III - CONVENTIONAL ESTIMATE START - ADVANTAGES

The preceding three sections have presented the mechanics for a manual solution of Phase III of the Critical Path Method beginning at any of three possible starting points. This section suggests certain advantages offered by the last procedure which begins with a schedule determined by conventional planning methods. Since this is a new approach for the Critical Path Method, it warrants a separate discussion.

The first advantage of the conventional estimate start is that CPM is made a more practical tool at the estimating stage of a contract. Since the estimator is usually working under time pressure, his primary goal is to determine feasible methods and costs early enough to enable his company to submit a proposal by the bidding deadline. In any remaining time, Phase III scheduling variations could be performed to improve the estimate. This assumes that there is time to prepare the Phase I network chart and time for the estimator to provide the additional time and cost data required. Ideally, CPM might be used in making the original estimate. A more logical stage of development, until CPM methods are improved and contractors are better acquainted with them, is to use them as secondary tools to improve the results of conventional estimates.

A second advantage of starting with a conventional estimate is that the final solution may not be far removed from the initial solution. A relatively few cycles of scheduling variation may give the ideal schedule for the specified project completion time. This tends to make the non-computer approach more practical. Moreover, as changes are made in project scheduling, the project time either remains at the specified time or varies only slightly from it. Since each change results in a cost improvement, the process may be discontinued at any point and still give a better solution for the desired project time.

A third advantage of starting with a conventional estimate is the most important and most debatable. The estimates of data for shortening or lengthening the various operations should be more realistic, and therefore more accurate, than estimates made in the other two approaches. This is based on the fact that the conventional estimate has established a realistic job pattern and setting. In general, methods and equipment for meeting the required schedule have been chosen. The approximate season of the year in which various operations are performed is known. Decisions regarding overtime hours, length of work week, and shift work have been made. All of these are subject to change by application of Phase III variations but it is unlikely that the overall pattern will change completely. In this approach the estimator is asked to consider each operation individually and to decide if it could be performed at a lower cost if more time were available and whether it could be performed faster if necessary. If so, he is asked to estimate both time and cost for the least cost method and for the crash time method. Any single scheduling change would represent a relatively minor variation from the condition of the original feasible estimate. A series of such changes might produce major differences in the job pattern but this can be handled effectively in a manual solution since the estimator has the opportunity to revise his figures at the end of any cycle. Such a procedure involves reasonable demands on the estimator and allows him to retain judgment control.

Contrast this with the requirements of a computer solution beginning at the project normal time-cost point. The estimator is first asked to compute the least cost of every operation assuming that the entire project would be performed at least direct cost. The fact that the resulting "normal" project duration may be twice as long as the allowable project duration means that the estimator is often figuring a completely different job which has very little semblance to the one to be bid. Next the estimator is asked to compute crash performance data for every operation. He attempts to visualize an "all-crash" project and to determine the cost of crashing each operation. Again he is dealing with an artificial job situation that is generally quite a bit different than the realistic conditions of the final solution. However, since all data is necessary as computer input and since there is no opportunity to revise the figures at intermediate steps of the procedure, the estimator must do the best that he can. He is being asked to estimate data for two extreme situations without any "feel" for the conditions of the final scheduling solution and, further, to assume a straight line variation of costs over the full time range between these extremes. It must be remembered that this input data is the basis for the output information and the resulting decisions. The less realistic it is, the less valuable will be the decisions.

It will be argued that cost-time variations for any operation can be estimated without considering the performance mode of other operations since CPM is based on the assumption that all operations are independent of one another except for sequential relationships. Unfortunately this is not true from a practical point of view. For example, crashing one operation may require labor crews to work multiple shift, 6 days per week. From a practical labor relations standpoint, crews on concurrent operations can hardly be kept on a 40-hour week. It will also be argued that starting at the end of the project time-cost curve is a more idealistic approach. It therefore is conceivable that this approach will lead to completely different results for a given project duration than are obtained by starting with the conventional estimate data. Certainly the estimator will be free from any bias if he does not consider the conditions surrounding a particular, feasible solution. While this argument may have some validity, the conventional estimate starting point is recommended since it is more likely to produce the most realistic final solution.

EVALUATION AND APPLICATION

This report has presented noncomputer procedures for applying all three phases of the Critical Path Method. In the introduction two justifications were given for the use of such procedures. One was the inconvenience of computer use for field organizations or small companies or, sometimes, a lack of familiarity with their use. A second was that the computer methods have certain shortcomings that can be overcome by a noncomputer approach. With a better understanding of the methods involved in CPM, these justifications can now be explained and evaluated more clearly.

Phase I of CPM is essentially a noncomputer procedure regardless of the manner in which the other phases are handled. The network relationships are developed, one by one, as they are represented graphically in chart form. As new relationships are recognized, additions and alterations are made on this chart. The arrow notation provides the dual numbering system for each operation that is required by the computer to establish the operation sequencing relationships. The noncomputer approach depends on visual observation of the network chart during various computational steps and, therefore, can be satisfied by the single numbering system of the simple circle notation. This circle notation makes Phase I easier by eliminating dummy operations and permitting the graphical representation of operation relationships in a more straightforward manner.

Phase II of CPM is a purely mechanical process once the network relationships between operations have been established and the time of performance of each operation has been estimated. Besides the network chart, only a single array of data is required, and the calculations consist of addition or subtraction of two numbers at a time. Even for a complex project, the noncomputer procedure is practical to apply. Therefore, the benefits of this phase are as available to the contractor who uses manual methods as to the one who employs a computer.

Phase III of CPM requires a complex and lengthy set of calculations even for a relatively simple example. It is for this phase that the computer has its most obvious advantages. The procedures offered in this report permit a noncomputer solution of such problems. However, a strong claim that the manual method is more convenient than the computer approach can hardly be made in view of the effort required to perform the manual computations. The advantage of the manual method is that it overcomes certain weaknesses of the computer solution which result from the assumption that each operation in the project is independent of every other operation except for the sequential relationships shown by the network chart. Given the normal time and cost of each operation, the crash time and cost of each operation, and the network relationships, the computer proceeds to grind out the project time-cost curve data and the data necessary to obtain corresponding schedules. This is exactly what was done also in the noncomputer procedure illustrated in this report. However, there is a basic difference. In the computer method it is impractical to exert intermediate control over the calculations to recognize changes in input data or related effects between operations. In the noncomputer method, this intermediate stage judgment control need not be relinquished. Changes may be made in the input data at the end of any cycle. Changes may also be made that jointly affect several operations to reflect the fact that changes in one operation often do affect other operations in ways not reflected by sequential relationships alone.

To illustrate how changes in one operation affect another, consider a project including the construction of a multi-span concrete trestle. On a conventional bar chart, the scheduling of the construction of this trestle might be shown as a continuous bar over a long duration of time. However, at certain stages of trestle completion other operations may begin. In this instance, CPM requires that this trestle work be broken down into several operations such as : "Build First X Spans," "Build Y Additional Spans," and "Build Last Z Spans." Suppose the Phase III analysis indicates the advisability of speeding up the operation "Build First X Spans." One way of speeding up this operation would be to use a different method or different equipment. However, from a practical standpoint, the method and equipment cannot be changed for the first group of spans without making the same change for the construction of the remaining spans In this case, a change in one operation affects two other operations.

Again, consider a slightly different plan. The operation "Build First X Spans" might be speeded up by working a 10-hour day instead of using different methods or equipment. In this case, the later two operations are not necessarily affected. However, there are other operations going on at the same time as the building of the first X spans. From a practical standpoint, it may not be possible to work a 10-hour shift on one operation without other crews working on concurrent operations also having a 10-hour day. Therefore, this change in one operation, although it does not affect subsequent operations does affect concurrent operations. A final alternative might be to speed up the operation "Build First X Spans" by simply adding more men to the crew for a temporary period of time. This change has a still different effect since it doesn't cause variation in any other operation. It is the only type of change that CPM theoretically recognizes.

In most construction work, changes in one operation more often than not affect other operations. Thus the cost slopes of many operations tend to change as the overall project is compressed in time. If one operation is shortened by adding an extra crane, say, to the job's equipment, then that operation has been charged with the move-in cost for the crane. Since the crane will be free part of the time and since it also can be used on other operations to shorten them, the cost slopes of the other operations will tend to decrease since they no longer have to pay the crane's move-in cost. The noncomputer method allows these cost slopes to be changed at this point. An alternative approach possible with the noncomputer method is to consider the overall results of adding one crane to the job. The time changes in all affected operations are computed and one cycle of shortening is analyzed to determine the corresponding time effect on the project. This project time change divided into the cost of moving in the crane and paying job rental for the necessary period of time gives a combined cost slope for the decision to use an additional crane. This cost slope may be compared to other possible cost slopes for shortening the job. This approach permits making the decision to use the additional crane before the point is reached that the shortening of a single operation can absorb the total move-in cost and justify its addition. The computer methods for CPM do not satisfactorily handle either of these approaches.

There are many decisions that the job planner must make in changing the project schedule that do affect several operations simultaneously in ways not indicated by the network chart. The decision to work a nine-hour day, to work a six-day week, to go to three-shift operations, to change the originally proposed methods, or to bring in different types or sizes of equipment are all examples of changes whose affects are not generally confined to a single operation. The Critical Path Method offers the planner the mechanics for determining the effect of any of these decisions. But it is only where the planner can proceed a step at a time and continually update his original data that he can most intelligently use these procedures. Unfortunately the computer methods are not as flexible in application. It is not practical to stop the computer at an intermediate step and to reappraise the input data in terms of the scheduling changes to that point.

Another planning opportunity is made possible by the noncomputer procedure and the opportunity to vary operation cost slopes. A good planner will prepare manpower and equipment requirement charts to correspond with his operation scheduling. It was mentioned earlier in the report that the determination of float times in Phase II gave a basis for shifting some operations schedulewise to take peaks and dips out of these manpower and equipment charts. To the extent that some peaks and dips still exist after noncritical operations have been shifted, further improvements are possible. Often the decision is made to carry a number of men or to retain pieces of equipment over a period when they are not required by the schedule. The cost of this labor or equipment rental might either be charged to overhead or to the operations that force this decision. In either case there are, for a given period, men or equipment available at zero cost. The operations in progress during this period should be examined to determine if they could be shortened further, at zero cost, by making use of this available manpower or equipment. Even ineffective use, if it served to provide any shortening, would be profitable. As a result, the schedule might be changed further to shorten project time, using zero cost slopes for certain operations. If project time had already been decreased to that desired, then certain other critical operations could be lengthened at a cost reduction to obtain a more favorable scheduling solution.

To sum up the evaluation of the noncomputer approach to the Critical Path Method, it might be said that where the necessary computation can be performed from start to finish in a purely mechanical fashion, the computer approach is much more desirable. On the other hand, where the mechanics of CPM can be coupled with the judgment of a skilled planner who is able to continually update data and experiment with changes involving more than one operation, then a superior project planning system is obtained by the manual method.

The Critical Path Method is founded on three basic assumptions. Each one is important, but, unfortunately, each one is seldom completely valid. The first such assumption is that the project network is a realistic model of project activity. It is very difficult to construct a project network chart that represents every important sequencing relationship properly. It is frequently easier to assume, for example, that one operation cannot commence until another is completed, when actually they may overlap and should be broken down differently. Unrealistic assumptions in constructing the network chart affect all other CPM calculations. The second assumption is that the linear time-cost relationship for each operation is a satisfactory approximation. This report suggests two advantages of the noncomputer procedure to improve this approximation. Using a feasible conventional estimate as a starting point, a more realistic setting is provided for making the initial approximation. Then, permitting adjustments of the cost slope data at the end of any cycle allows the approximation to be further improved as the schedule takes shape. The third assumption is that all operations in the network are independent of one another except for network sequencing relationships. This is generally not true. An advantage of the noncomputer approach is that it permits proper consideration to be given to changes that do affect more than one operation.

Practically all engineering methods depend on assumptions involving approximations. The effectiveness of these methods is measured by the dependability with which results may be successfully applied. CPM is no exception. If after a solution is obtained and final adjustments are made, CPM gives consistently better results than those obtained by trial and error or judgment alone, then the assumptions are justified.

FUTURE IMPLICATIONS

The Critical Path Method is already an accepted tool of industry. Its uses for planning, scheduling, and control have been indicated in this report. In closing, a few predictions will be made concerning possible future developments.

(1) The use of CPM in connection with equitable settlements of claims for change orders and extra work orders has been mentioned. Not only does CPM provide the mechanics for determining all of the operations affected by a given change as well as the effect on project duration, but it provides the mechanics for determining the least cost to compress project completion time back to its original date if this is necessary. As jobs become more complex and time limits more critical, as in the case of missile-base construction, CPM will probably be used more extensively by contractors to document claims. It is already being employed for just this purpose.[1] In turn, it will become necessary for owner representatives to understand CPM and, possibly, demand submission of project network charts as part of the contract requirements in order to protect their interests. Understanding CPM for this purpose implies more than mere acquaintance wituh the input data and output data from computer programmed solutions. A basic knowledge of the assumptions and approximations on which CPM is based will be necessary.

(2) An increasing use of CPM will tend to place more importance on performance time data as well as performance cost data in contractors' cost records. Conventional cost accounting provides unit costs for the various operations that comprise the project. Usually there is little or no documentation to indicate relative speed of performance. On the other hand, CPM recognizes that costs vary with the time allowed and that a crash operation will have considerably higher costs than one performed at the normal production rate. A CPM schedule requires project operations to be performed at various rates ranging from normal to crash. It is conceivable that cost figures provided to management for control and to engineers for future estimating may carry an accompanying indication as to whether the

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[1] "Missile-Base Builder Wins Profit With Paper," Engineering News-Record, September 7, 1961.

operation was performed at "normal production speed," "25% crash," or "100% crash" for example. But the solution is not this simple. The point to be made here is that the intelligent use of job data for CPM planning, estimating, or control will require some system of documentation to reflect actual performance rates as well as actual costs.

(3) A rational approach to decisions regarding expenditures to lessen risks is possible with CPM. For example, the probability that the river level on a dam project will reach the cofferdam crest elevation at various dates might be determined from hydrographic data. The cost of the resulting damage at these dates could be estimated. The probability multiplied by the cost would indicate the "cost of the risk." CPM in turn provides a method for computing the cost of speeding up the critical operations to decrease this risk cost. A balance between the cost of further project speed-up and the rate at which the cost of risk is being decreased would provide another basis for scheduling. A similar approach might be taken in avoiding liquidated damages charges on a project having a time limit specified in calendar days. In the given number of calendar days, the probabilities of having various numbers of working days may be estimated. CPM could be used to determine the cost of shortening the job in this range, and the rate of cost increase could be compared to the rate at which the cost of the risk is decreased to obtain a logical scheduling balance point. Such studies on a long duration project could be repeated periodically as the probabilities involved were changed.

(4) From the contractor's viewpoint CPM provides a method for determining a schedule for meeting the owner's completion date at least cost. In some cases the owner offers an incentive in the form of a bonus clause if the contractor betters this completion time. In the absence of such an incentive, there still may be some reasons to aim for earlier completion. As mentioned earlier in the report, the contractor's ever increasing overhead cost curve can be plotted against the project time-direct cost curve obtained by CPM Phase III. A balance between the cost of speeding up the project and the rate at which indirect costs are increasing may indicate the desirability of earlier completion. The cost of lessening certain risks may also influence the scheduling, as just discussed. Many

intangibles may also be involved such as releasing key men for other work or winning the respect of the owner by shortening the schedule. Presently, however, there seldom seems to be sufficient incentive to make early completion attractive to contractors. An executive of a chain of grocery stores recently complained that completion dates of similar buildings varied from 55 days to over 200 days. In such cases, there is certainly a dollar value to the owner from earlier completion. Even for public projects, early completion represents dollars savings. Many competent contractors could provide such savings if given a slight incentive to do so. The use of CPM in their job planning would be an additional tool for this purpose. A bidding system that permits contractors to bid completion time as well as contract amount seems desirable. Some formula, established in advance, for converting bids to a comparable basis and for establishing liquidated damages would be necessary. The point is that owners need a system for providing a realistic incentive for completion at that point in time that makes the total cost of the structure, including the value of early completion, the least. With such incentives, CPM can provide a more effective tool for cost reduction.

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APPENDIXES
OF
CHARTS AND TABLES

APPENDIX A

PHASE I

PROJECT NETWORK

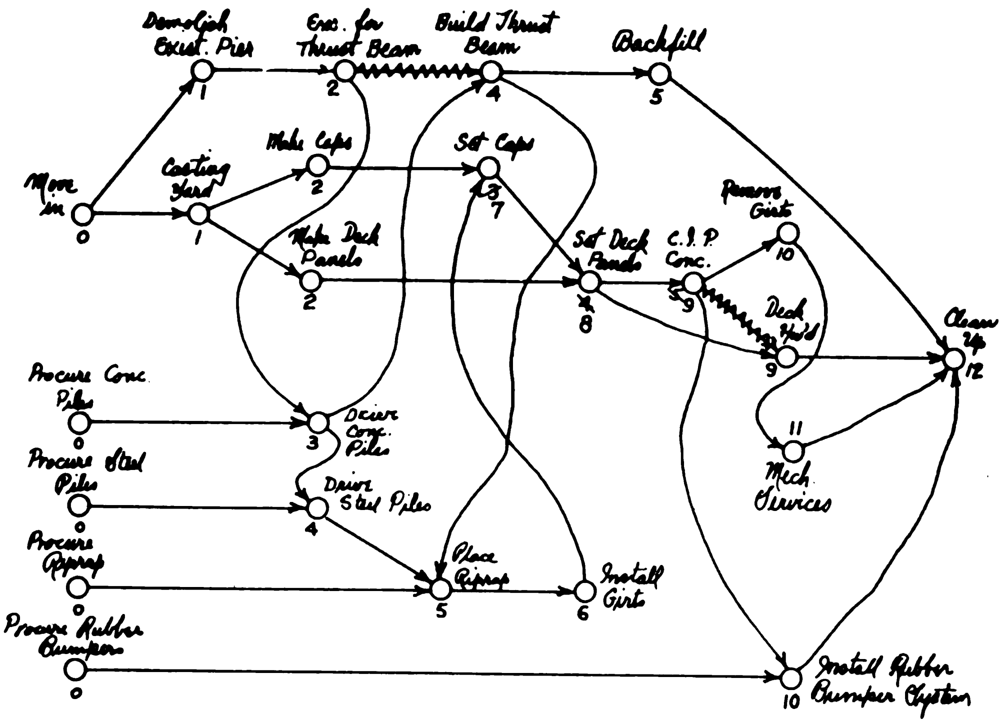


Figure 1

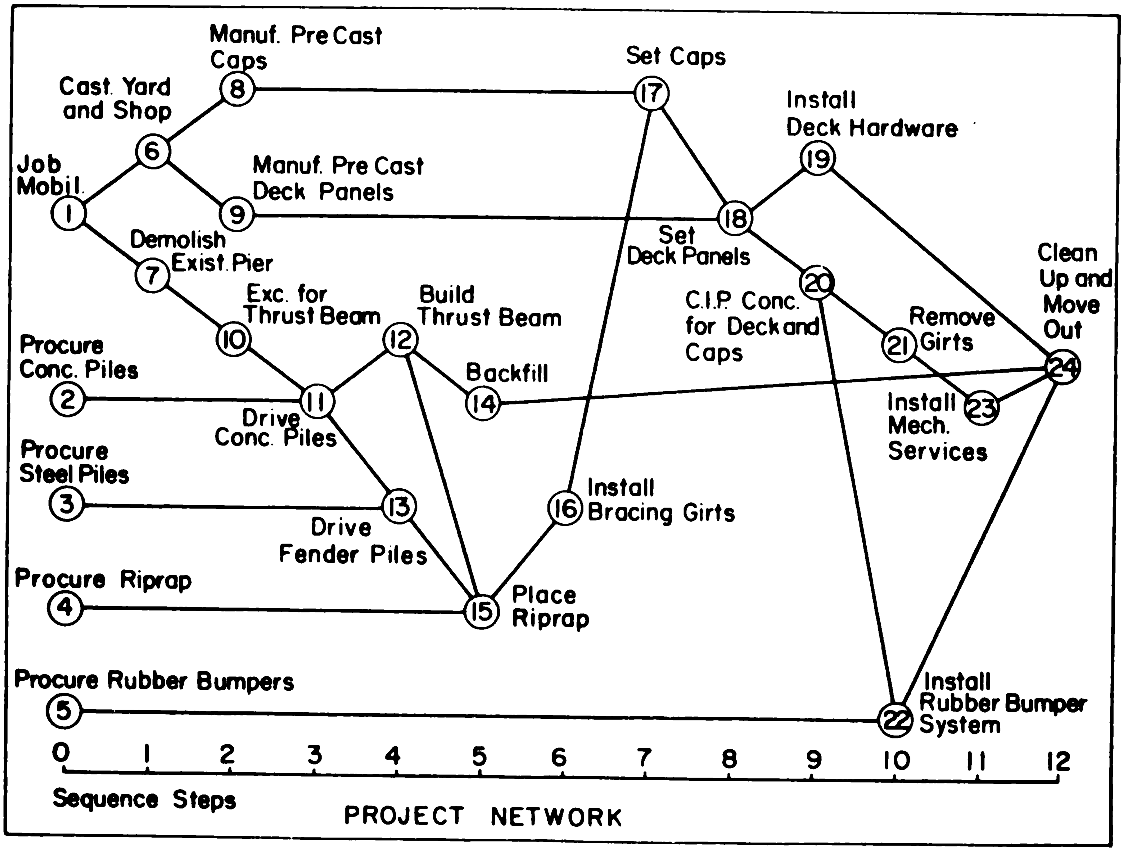


Figure 2

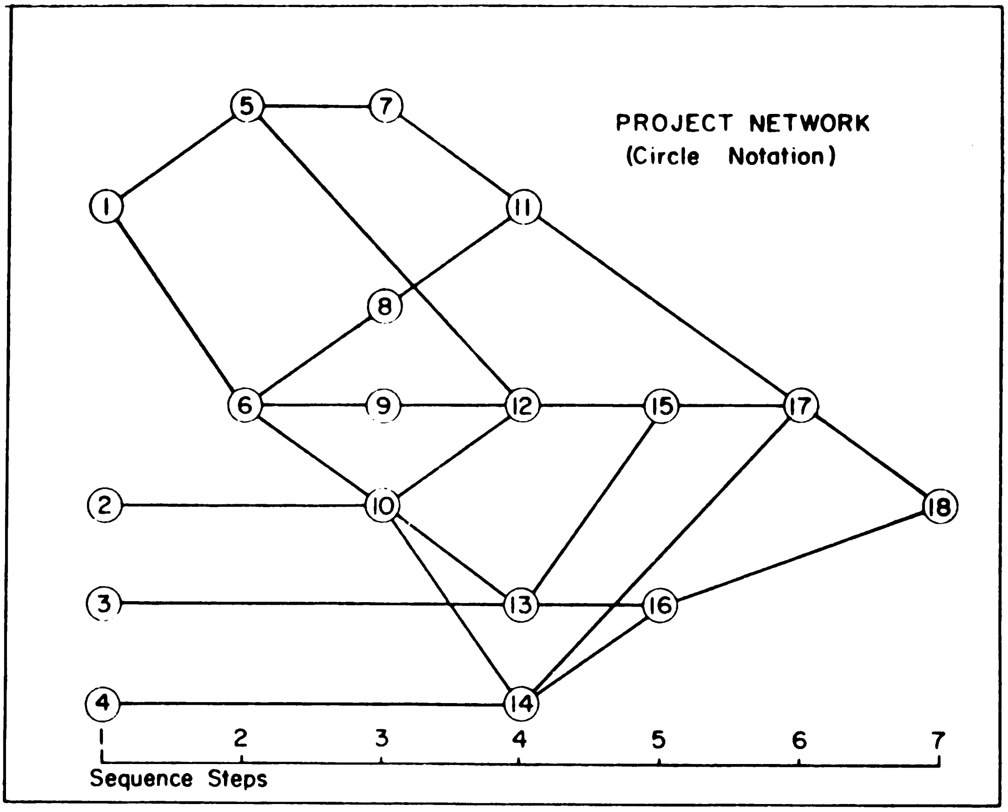


Figure 3

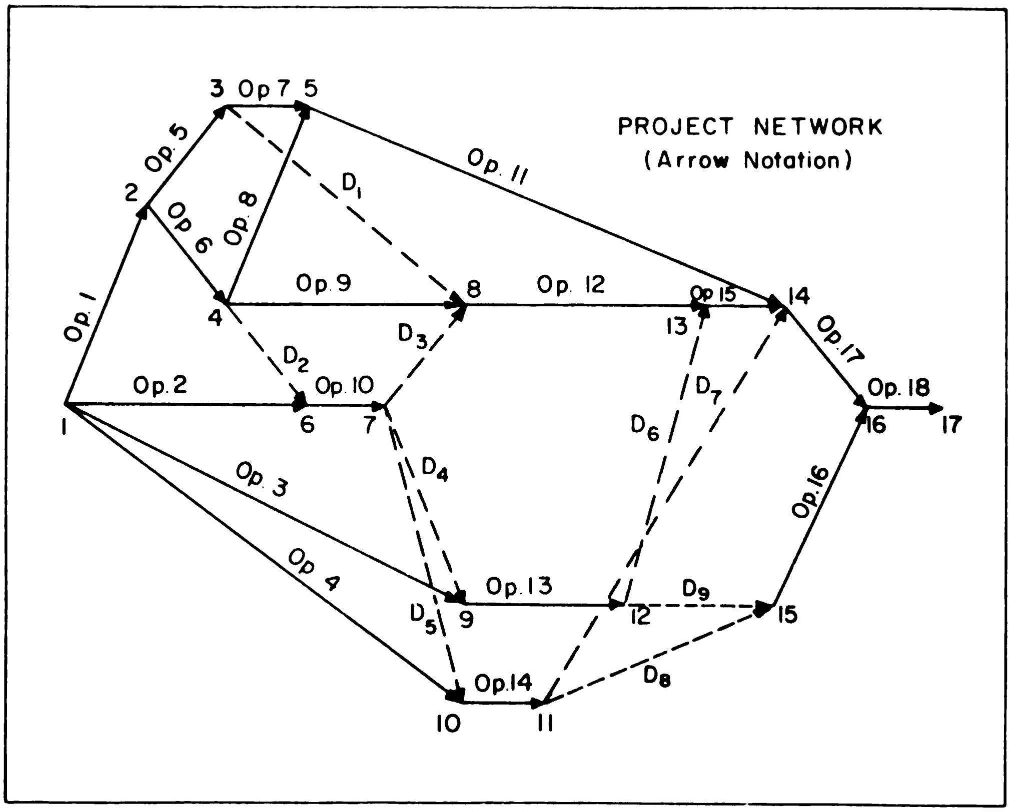


Figure 4

APPENDIX B

PHASE II
CONSTRUCTION SCHEDULE

DETERMINATION OF CRITICAL OPERATIONS, TIME BOUNDARIES, AND FLOATS

Op	Est. Time	Start		Finish		Float		Crit. Op.	
		Earliest	Latest	Earliest	Latest	Total	Free		
1	5	0		5					
2	15	0		15					
3	30	0		30					
4	20	0		20					
5	12	5		17					
6	6	5		11					
7	24	17		41					
8	8	11		19					
9	4	11		15					
10	10	15		25					
11	11	41		52					
12	9	25		34					
13	14	30		44					
14	21	25		46					
15	10	44		54					
16	12	46		58					
17	7	54		61					
18	3	61		64					

Figure 5A

DETERMINATION OF CRITICAL OPERATIONS, TIME BOUNDARIES, AND FLOATS

Op.	Est. Time	Start		Finish		Float		Crit. Op.	
		Earliest	Latest	Earliest	Latest	Total	Free		
1	5		2		7				
2	15		3		18				
3	30		0		30				
4	20		8		28				
5	12		7		19				
6	6		12		18				
7	24		19		43				
8	8		35		43				
9	4		31		35				
10	10		18		28				
11	11		43		54				
12	9		35		44				
13	14		30		44				
14	21		28		49				
15	10		44		54				
16	12		49		61				
17	7		54		61				
18	3		61	64 →	64				

Figure 5B

DETERMINATION OF CRITICAL OPERATIONS, TIME BOUNDARIES, AND FLOATS									
Op	Est. Time	Start		Finish		Float		Crit. Op.	
		Earliest	Latest	Earliest	Latest	Total	Free		
1				5	7	2			
2				15	18	3			
3				30	30	0		✓	
4				20	28	8			
5				17	19	2			
6				11	13	2			
7				41	43	2			
8				19	43	24			
9				15	35	20			
10				25	28	3			
11				52	54	2			
12				34	44	10			
13				44	44	0		✓	
14				46	49	3			
15				54	54	0		✓	
16				58	61	3			
17				61	61	0		✓	
18				64	64	0		✓	

Figure 5C

DETERMINATION OF CRITICAL OPERATIONS, TIME BOUNDARIES, AND FLOATS									
Op.	Est. Time	Start		Finish		Float		Crit. Op.	
		Earliest	Latest	Earliest	Latest	Total	Free		
1		0		5			0		
2		0		15			0		
3		0		30			0		
4		0		20			5		
5		5		17			0		
6		5		11			0		
7		17		41			0		
8		11		19			22		
9		11		15			10		
10		15		25			0		
11		41		52			2		
12		25		34			10		
13		30		44			0		
14		25		46			0		
15		44		54			0		
16		46		58			3		
17		54		61			0		
18		61		64			0		

Figure 5D

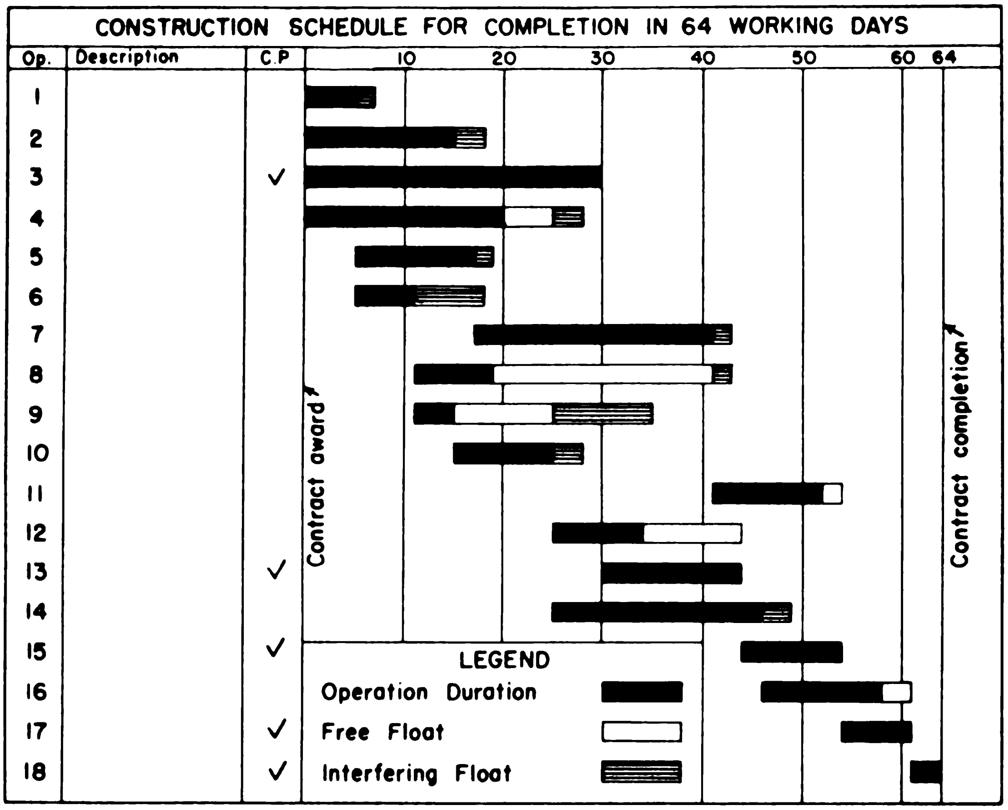


Figure 6

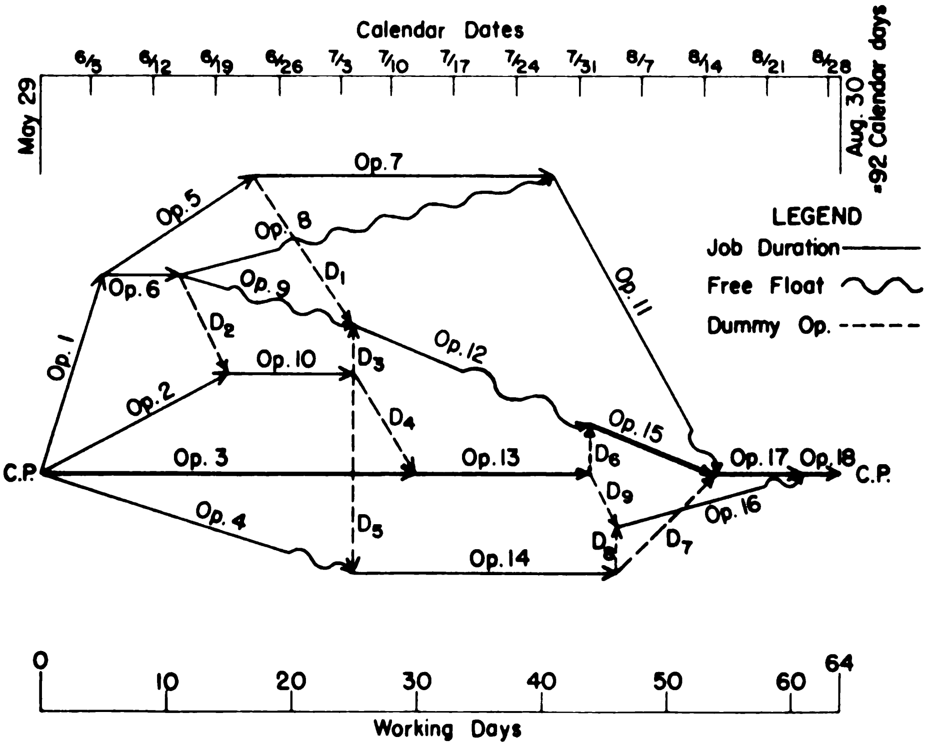


Figure 7

APPENDIX C

PHASE III

SCHEDULING VARIATIONS - GENERAL

PROJECT TIME-COST RELATIONSHIPS

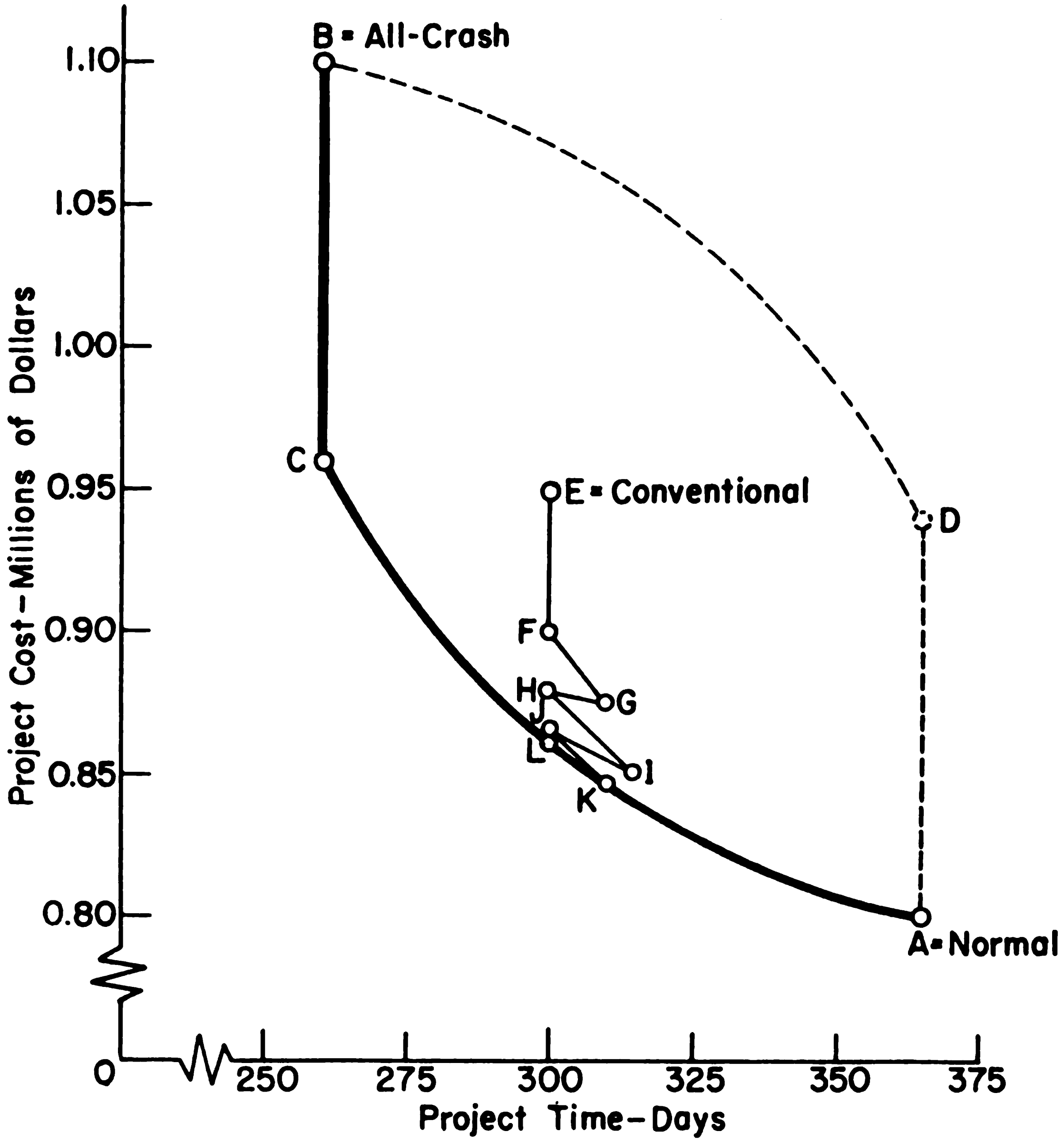


Figure 8

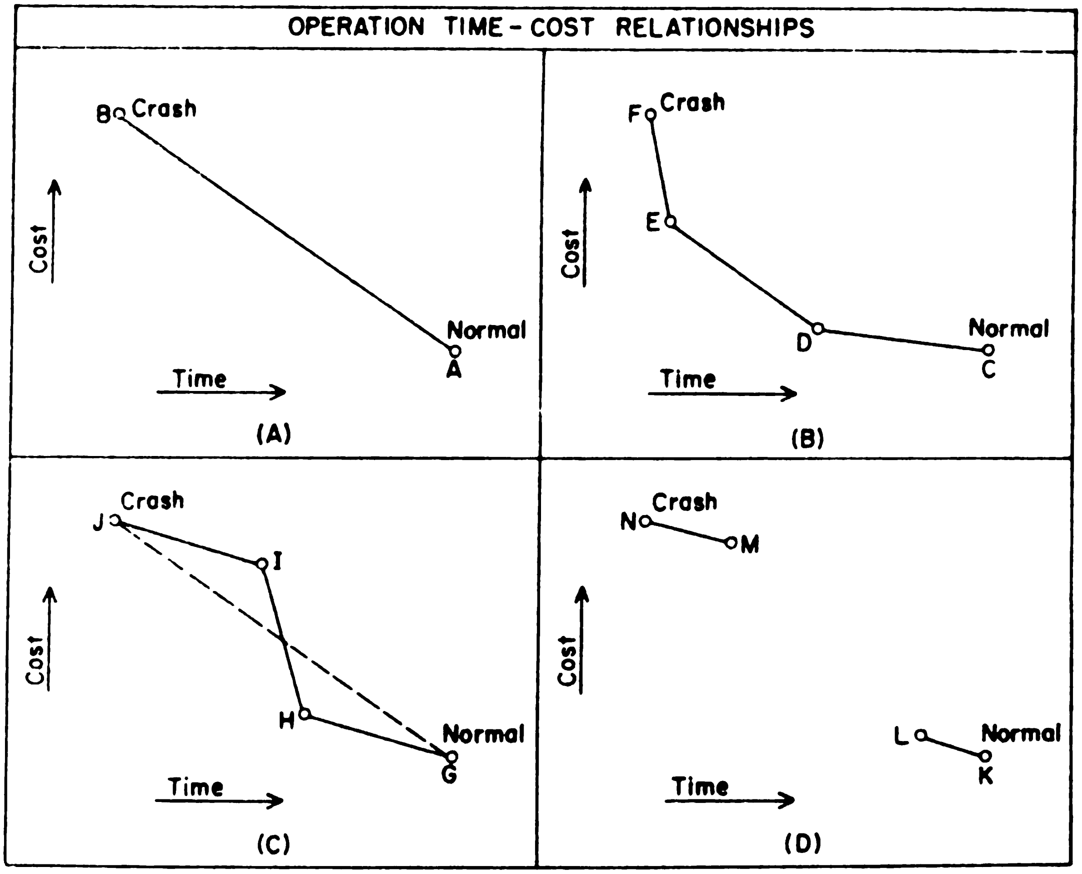

OPERATION TIME - COST RELATIONSHIPS
B
Crash
Cost
Time
Normal
A
(A)
F
Crash
E
Cost
D
Time
Normal
C
(B)
J
Crash
I
Cost
H
Time
Normal
G
(C)
N
Crash
M
Cost
L
Normal
K
Time
(D)

Figure 9

APPENDIX D

PHASE III
"NORMAL START" PROCEDURE

TIME AND COST DATA - SLOPE DETERMINATION - PROJECT CRASH TIME

Op	Normal Prod. Time	Normal Prod. Cost	Crash Prod. Time	Crash Prod. Cost	Difference Time	Difference Cost	Slope $ / Day	Crash Schedule Earl. Start	Crash Schedule Earl. Fin
1	5	1500	5	1500	0	0	none	0	5
2	15	7200	10	8000	5	800	160	0	10
* 3	30	8400	18	9000	12	600	50	0	18
4	20	2100	14	2700	6	600	100	0	14
5	12	1400	8	1560	4	160	40	5	13
6	6	800	4	1200	2	400	200	5	9
7	24	6800	20	7800	4	1000	250	13	33
8	8	1000	5	1240	3	240	80	9	14
9	4	600	3	900	1	300	300	9	12
10	10	3000	7	3450	3	450	150	10	17
11	11	2500	8	3580	3	1080	360	33	41
12	9	1800	6	2700	3	900	300	17	23
* 13	14	2600	10	3320	4	720	180	18	28
14	21	8400	15	10,800	6	2400	400	17	32
* 15	10	1900	6	2140	4	240	60	28	34
16	12	1300	10	1400	2	100	50	32	42
* 17	7	700	5	840	2	140	70	41	46
* 18	3	500	3	500	0	0	none	46	⑭⑨
	Total	52,500	Total	62,630					

64 Figure 10

SUMMARY SHEET - PROJECT SCHEDULING ADJUSTMENTS

Cycle	Min. Op. Slope Comb. # 1	# 2	# 3	# 4	Op. Poss. Shortening # 1	# 2	# 3	# 4	Interact Limit	Days Change	Cost / Day	Cost Change	Total Project Cost	Total Project Days
0													52,500	64
1	3				12				2	2	50	100	52,600	62
2	17				2				1	1	70	70	52,670	61
3	16	17			2	1			none	1	120	120	52,790	60
4	3	5	16		10	4	1		3	1	140	140	52,930	59
5	3	5	10		9	3	3		5	3	240	720	53,650	56
6	2	3	7		5	6	4		2	2	460	920	54,570	54
7	2	3	4	7	3	4	6	2	2	2	560	1120	55,690	52
8	3	11	14		2	3	6		2	2	810	1620	57,310	50
9	11	14	15		1	4	4		5	1	820	820	58,130	49
10	none													

Figure 11

OPERATION SELECTION AND TIME TALLY SHEET

Op	Slope $/Day	C P Cycle	Finish Cycle	Normal Time	Poss Short.	Revised Time / Remaining Possible Shortening									
						Cycle 1	2	3	4	5	6	7	8	9	10
5	40	1	5	12	4				11 3	8 0					
3	50	0	8	30	12	28 10			27 9	24 6	22 4	20 2	18 0		
16	50	2	4	12	2			11 1	10 0						
15	60	0	9NC	10	4									9 3	
17	70	0	3	7	2		6 1	5 0							
8	80			8	3										
4	100	6	9NC	20	6							18 4			
10	150	2	5	10	3					7 0					
2	160	2	9NC	15	5						13 3	11 1			
13	180	0	9NC	14	4										
6	200	7	9NC	6	2										
7	250	1	7	24	4						22 2	20 0			
9	300			4	1										
12	300			9	3										
11	360	1	9	11	3								9 1	8 0	
14	400	2	9NC	21	6								19 4	18 3	
1	none	1	0	5	0										
18	none	0	0	3	0										

Figure 12

W. Short no 2

NETWORK INTERACTION LIMIT DETERMINATION

Cycle →					1		2		3		4		5		6		7		8		9			
Operations Shortened →					3		17		16, 17		3, 5, 16		3, 5, 10		2, 3, 7		2, 3, 4, 7		3, 11, 14		11, 14, 15			
Interaction Limit →					2		1		none		3		5		2		2		2		5			
Days Shortened →						2		1		1		1		3		2		2		2		1		
Pre. Op.	Post Op.	Pre. E.F.	Post E.S.	Log		Log		Log		Log		Log		Log		Log		Log		Log		Log		
1	5	5	5	0																				
	6	5	5	0																				
2	10	15	15	0											< <		< <							
3	13	30	30	0	< <						< <		< <		< <		< <		< <					
4	14	20	25	5									(<)	2	(<)	0	< <							
5	7	17	17	0							< <		< <											
	12	17	25	8							<	9	< <		<	7	<	5						
6	8	11	11	0																				
	9	11	11	0																				
	10	11	15	4											<	2	(<)	0						
7	11	41	41	0							< <		< <		< <		< <							
8	11	19	41	22							<	21	<	18	<	16	<	14						
9	12	15	25	10									<	7	<	5	<	3						
10	12	25	25	0									< <		< <		< <							
	13	25	30	5	<	3					(<)	2	< <		< <		< <		(<)	0				
	14	25	25	0									< <		< <		< <							
11	17	52	54	2	(<)	0					< <		< <		< <		< <		< <		< <			
12	15	34	44	10	<	8					<	7	< <		< <		< <		<	5				
13	15	44	44	0	< <						< <		< <		< <		< <		< <					
	16	44	46	2	<	4					<	5	< <		< <		< <		< <		(<)	4		
14	16	46	46	0									< <		< <		< <		< <		< <			
	17	46	54	8	<	6					<	5	< <		< <		< <		< <		< <			
15	17	54	54	0	< <						< <		< <		< <		< <		< <		< <			
16	18	58	61	3	<	1	(<)	0	< <		< <		< <		< <		< <		< <		< <			
17	18	61	61	0	< <		< <		< <		< <		< <		< <		< <		< <		< <			

ADD

SUB

Figure 13

PRECEDENCE MATRIX

Post Ops

Pre-Ops.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1					O	O														
2										O										
3													O							
4																				
5							O													
6								O	O											
7											O									
8																				
9																				
10												O		O						
11																				
12																				
13															O					
14																O				
15																	O			
16																				
17																		O		
18																				
19																				
20																				

Figure 14A

PRECEDENCE MATRIX

Start

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1					O	O	X	X	X		X									
2										O		X		X		X				
3													O		X		X	X		
4																				
5							O				X									
6								O	O											
7											O									
8																				
9																				
10												O		O		X				
11																				
12																				
13															O		X	X		
14																O				
15																	O	X		
16																				
17																		O		
18																				
19																				
20																				

Figure 14B

PRECEDENCE MATRIX
End of Cycle 1

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1					O	O	X	X	X		X						X	X		
2										O		X		X		X				
3													O		X		X	X		
4																				
5							O				X						X	X		
6								O	O											
7											O						X	X		
8																				
9																				
10												O		O		X				
11																	O	X		
12																				
13															O		X	X		
14																O				
15																	O	X		
16																				
17																		O		
18																				
19																				
20																				

Figure 14C

PRECEDENCE MATRIX
End of Cycle 2

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1					O	O	X	X	X		X						X	X		
2										O		X		X		X		X		
3													O		X		X	X		
4																				
5							O				X						X	X		
6								O	O											
7											O						X	X		
8																				
9																				
10												O		O		X		X		
11																	O	X		
12																				
13															O		X	X		
14																O		X		
15																	O	X		
16																		O		
17																		O		
18																				
19																				
20																				

Figure 14D

PRECEDENE MATRIX
End of Cycle 6

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1					O	O	X	X	X		X						X	X		
2										O		X		X		X		X		
3													O		X		X	X		
4														O		X		X		
5							O				X						X	X		
6								O	O											
7											O						X	X		
8																				
9																				
10												O		O		X		X		
11																	O	X		
12																				
13															O		X	X		
14																O		X		
15																	O	X		
16																		O		
17																		O		
18																				
19																				
20																				

Figure 14E

PRECEDENCE MATRIX
End of Cycle 7

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1					O	O	X	X	X	X	X	X		X		X	X	X		
2										O		X		X		X		X		
3													O		X		X	X		
4														O		X		X		
5							O				X						X	X		
6								O	O	O		X		X		X		X		
7											O						X	X		
8																				
9																				
10												O		O		X		X		
11																	O	X		
12																				
13															O		X	X		
14																O		X		
15																	O	X		
16																		O		
17																		O		
18																				
19																				
20																				

Figure 14F

PRECEDENCE MATRIX
End of Cycle 8

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1					○	○	X	X	X	X	X	X	$\boxed{X}$	X	$\boxed{X}$	X	X	X		
2										○		X		X		X		X		
3													○		X		X	X		
4														○		X		X		
5							○				X						X	X		
6								○	○	○		X	$\boxed{X}$	X	$\boxed{X}$	X	$\boxed{X}$	X		
7											○						X	X		
8																				
9																				
10												○	$\boxed{○}$	○	$\boxed{X}$	X	$\boxed{X}$	X		
11																	○	X		
12																				
13															○		X	X		
14																○		X		
15																	○	X		
16																		○		
17																		○		
18																				
19																				
20																				

Figure 14G

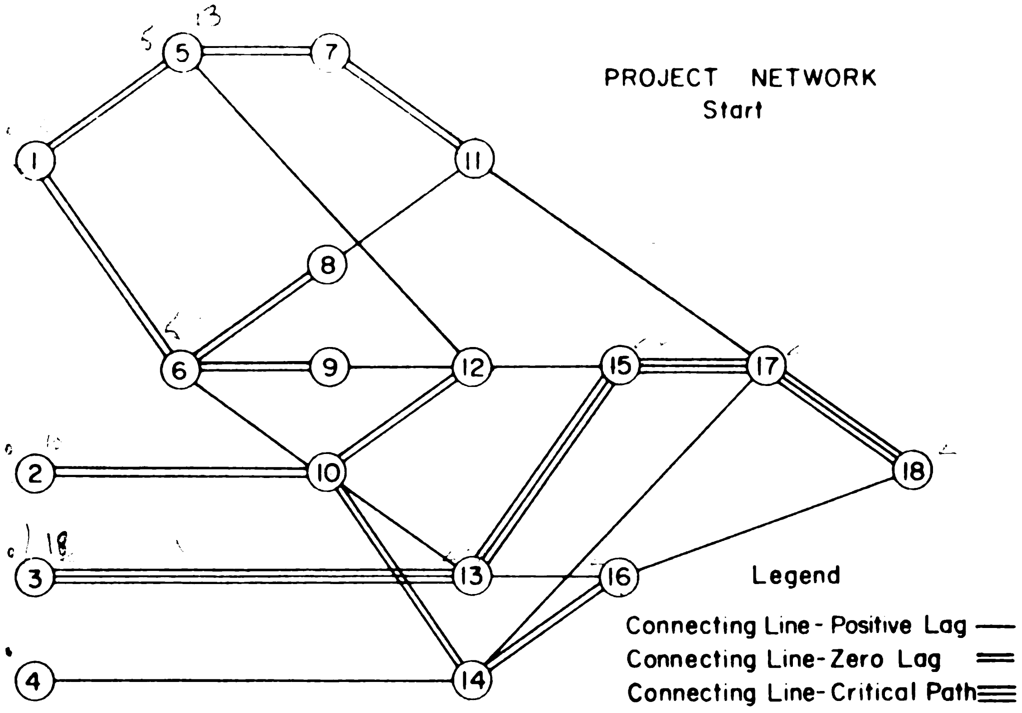


Figure 15A

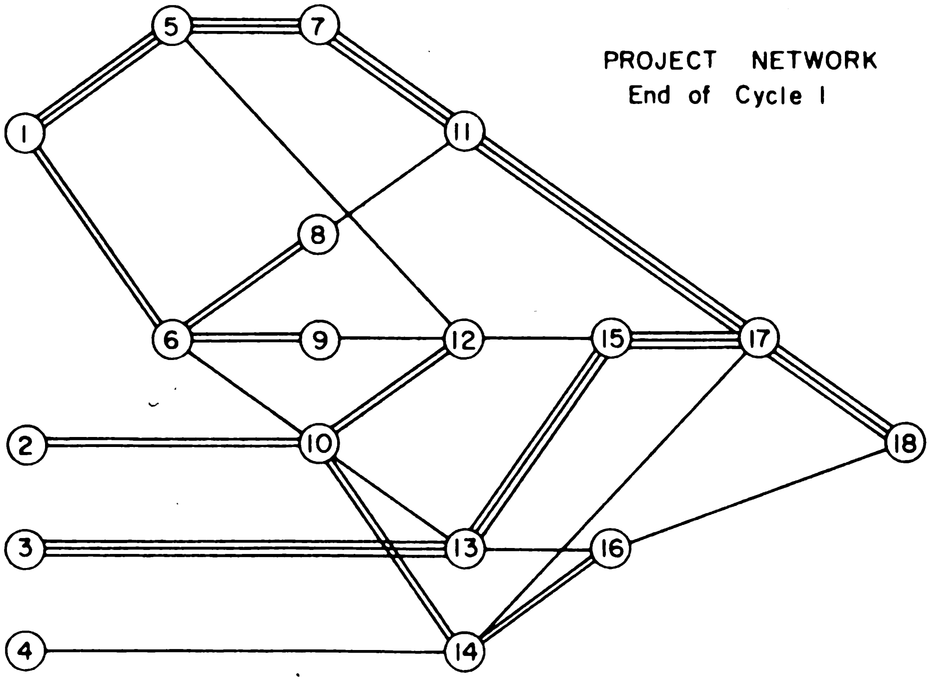


Figure 15B

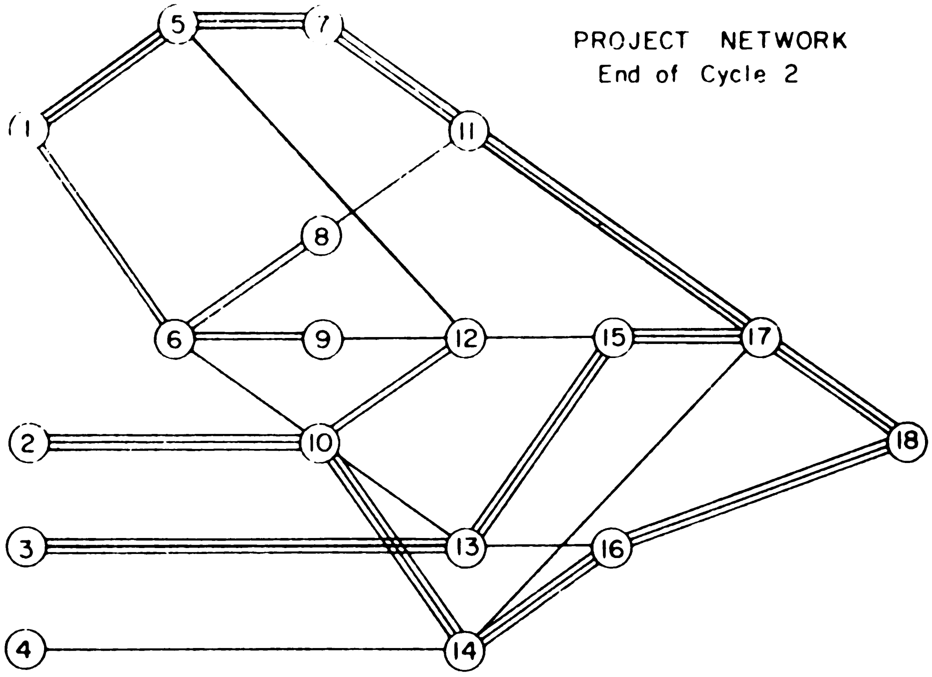


Figure 15C

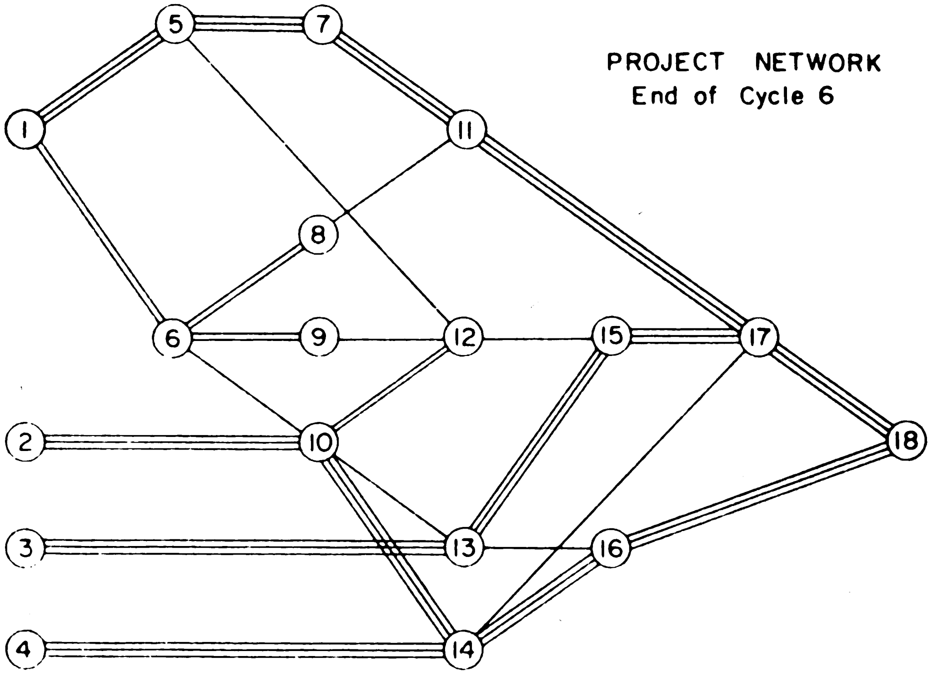


Figure 15D

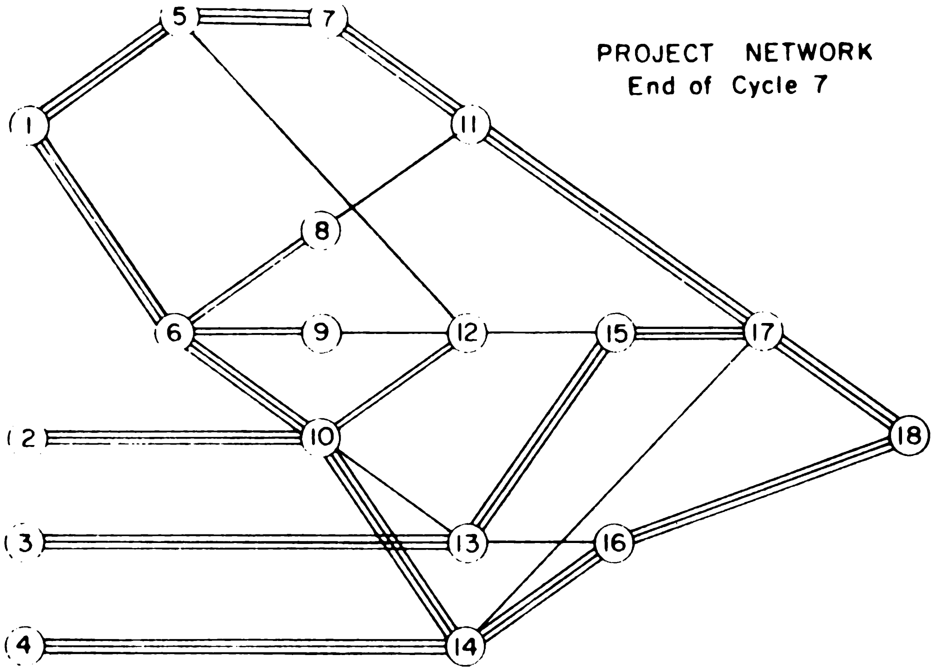

PROJECT NETWORK
End of Cycle 7

Figure 15E

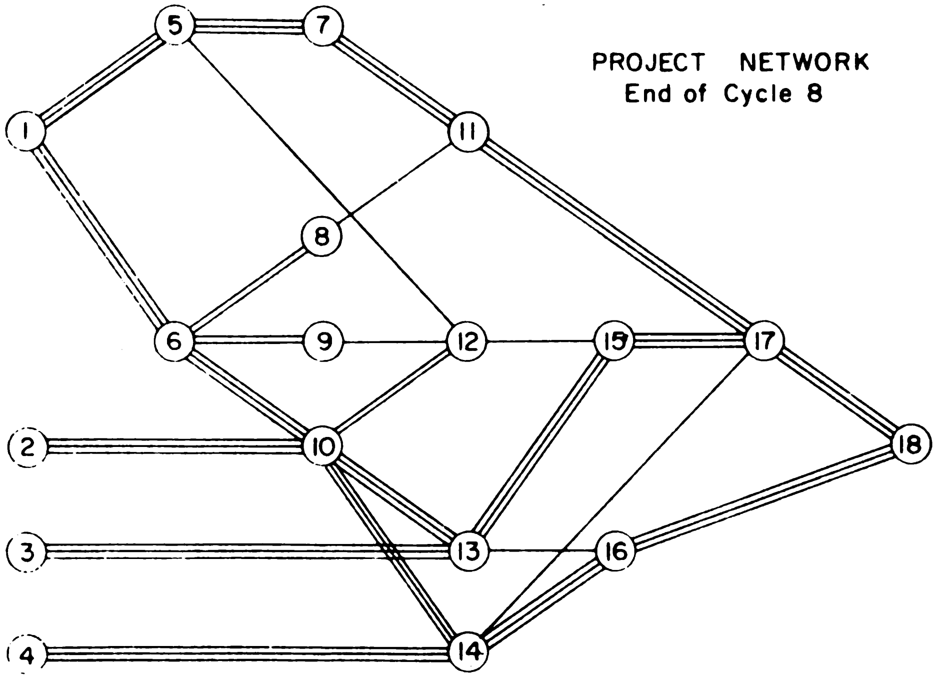

PROJECT NETWORK
End of Cycle 8

Figure 15F

DETERMINATION OF MINIMUM OPERATION SLOPES COMBINATION

Oper. Sequence		Cycle 1	
1-4		③	1
5-6		-	⑤
7-10		-	7
11-14		13	11
15-16		15	-
17			
18			

Op.	Slope	Cycle 2		
5	40	X	X	
3	50	X		
16	50			
15	60		X	
17	70			X
8	80			
4	100			
10	150			
2	160			
13	180			
6	200			
7	250			
9	300			
12	300			
11	360			
14	400			
		90	X	(70)

Figure 16A

DETERMINATION OF MINIMUM OPERATION SLOPES COMBINATION

Operation Sequence		Cycle 1	Cycle 2	Cycle 6	Cycle 7	Cycle 8	
1 - 4		(~~3~~) ~~1~~	~~3~~ (2)	(2) (4)	(2) ~~1~~	~~3~~ 2	Note Following Cycle 9, the remaining operation numbers may be struck out See report text explanation
5 - 6		- (~~5~~)	-	- -	(6)	-	
7 - 10		(~~7~~)	- (~~10~~)	~~10~~ -		~~10~~	
11 - 14		13 (~~11~~)	13 14				
15 - 16		(15) -	(15) (~~16~~)				
17			~~17~~ -				
18							

Op	Slope		Cycle 2	Cycle 3	Cy 4	Cycle 5	Cycle 6	Cycle 7	Cycle 8	Cycle 9
5	40		X X	X X	X X	X X X X				
3	50		X	X	X	X X	X X X	X X X	X X	
16	50			X X X	X X					
15	60		X	X	X	X	X	X	X	X X
17	70		X	X						
8	80									
4	100							X X X X	X X X	X X
10	150					X X X				
2	160					X	X X X X	X X X X	X X X	X X
13	180					X	X	X	X	X
6	200								X X X	X X
7	250						X X X X	X X X X		
9	300									
12	300									
11	360						X	X	X X X X	X X X
14	400						X	X	X	X
			30 x 70	140 x 120	(140) x	(240) x x x	(460) x x x x	(560) x x x 700	870 x x (810)	880 x (820)

Figure 16b

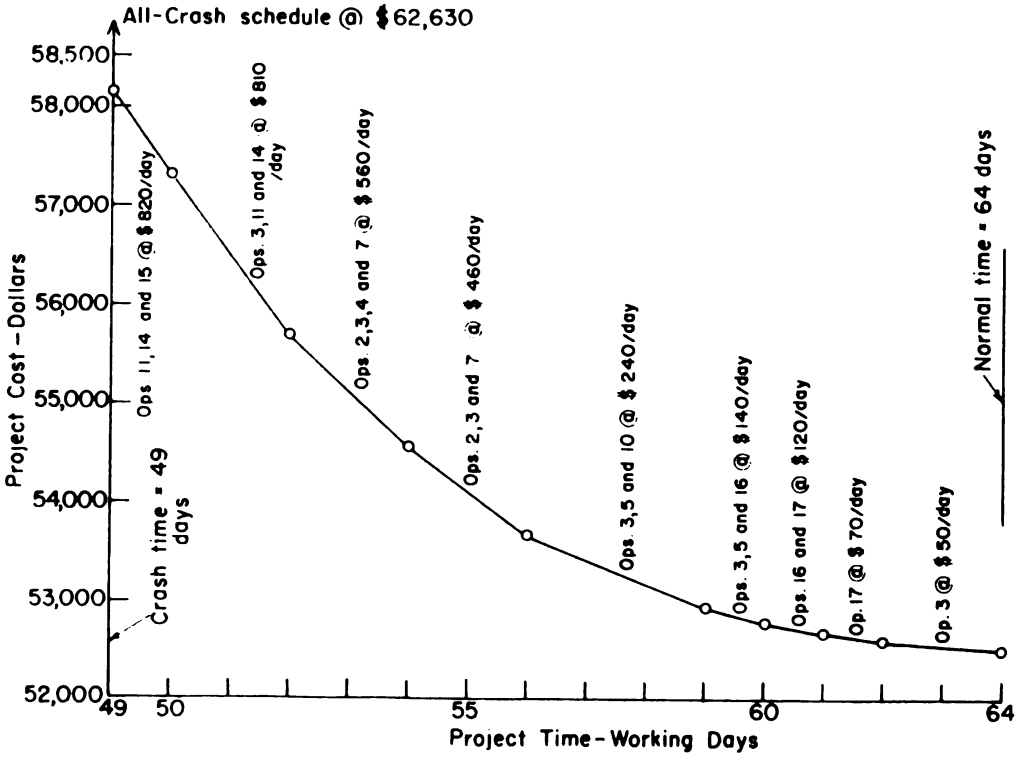


Figure 17

APPENDIX E

PHASE III
"ALL-CRASH START" PROCEDURE

SUMMARY SHEET - PROJECT SCHEDULING ADJUSTMENTS															
Cycle	Max Op Slope Comb				Op Poss Lengthening				Interact Limit		Days Change	Cost/Day	Cost Change	Total Project Cost	Total Project Days
	# 1	# 2	# 3	# 4	# 1	# 2	# 3	# 4							
0										↑ Non-Critical Lengthening ↓				62,630	49
1	14				6				4		4	400	1600	61,030	49
2	9				1				5		1	300	300	60,730	49
3	12				3				5		3	300	900	59,830	49
4	6				2				1		1	200	200	59,630	49
5	13				4				7		4	180	720	58,910	49
6	4				6				3		3	100	300	58,610	49
7	8				3				18		3	80	240	58,370	49
8	15				4				3		3	60	180	58,190	49
9	2	4	6	11/15	5	3	1	3/1	1	↑ Critical Lengthening ↓	1	880	880	57,310	50
10	3	11	14		12	2	2		none		2	810	1620	55,690	52
11	2	3	4	7	4	10	2	4	none		2	560	1120	54,570	54
12	2	3	7		2	8	2		none		2	460	920	53, 650	56
13	3	5	10		6	4	3		none		3	240	720	52,930	59
14	3	5	16		3	1	2		5		1	140	140	52,790	60
15	16	17			1	2			none		1	120	120	52,670	61
16	17				1				none		1	70	70	52, 600	62
17	3				2				4		2	50	100	52, 500	64

Figure 18

OPERATION SELECTION AND TIME TALLY SHEET																							
Op.	Slope $/Day	C. P Cycle	Finish Cycle		Crash Time	Poss Len	Revised Time / Remaining Possible Lengthening																
							Cy. 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
14	400	X	10		15	6	19 2									21 0							
11	360	0	10		8	3									9 2	11 0							
9	300		2		3	1		4 0															
12	300		3		6	3			9 0														
7	250	0	12		20	4											18 2	20 0					
6	200	~~4~~	9		4	2				5 1					6 0								
13	180	8	5		10	4					14 0												
2	160	X	12		10	5									11 4		13 2	15 0					
10	150	X	13		7	3													10 0				
4	100	~~8~~	11		14	6						17 3			18 2		20 0						
8	80		7		5	3							8 0										
17	70	0	16		5	2															6 1	7 0	
15	60	8	9		6	4								9 1	10 0								
3	50	8	17		18	12										20 10	22 8	24 6	27 3	28 2			30 0
16	50	X	15		10	2														11 1	12 0		
5	40	0	14		8	4													11 1	12 0			
1	none	0	0		5	0																	
18	none	0	0		3	0																	

Figure 19

NETWORK INTERACTION LIMIT DETERMINATION - SHEET 1

Cycle →					1		2		3		4		5		6		7		8		9		10	
Operations Lengthened →					14		9		12		6		13		4		8		15		2,4,6 11,15		3,11 14	
Interaction Limit →					4		5		5		1		7		3		18		3		1		none	
Days Lengthened →						4		1		3		1		4		3		3		3		1		2
Pre Op	Post Op	Pre EF	Post ES	Log		Log		Log		Log		Log		Log		Log		Log		Log		Log		Log
1	5	5	5	0																				
	6	5	5	0																				
2	10	10	10	0																	> >			
3	13	18	18	0																			> >	
4	14	14	17	3											>	0					> >			
5	7	13	13	0																				
	12	13	17	4																	>	5		
6	8	9	9	0							> >										> >			
	9	9	9	0							> >										> >			
	10	9	10	1							>	0									> >			
7	11	33	33	0																				
8	11	14	33	19							>	18					>	15			>	14		
9	12	12	17	5			>	4			>	3									> >			
10	12	17	17	0																	> >			
	13	17	18	1																	>	0	>	2
	14	17	17	0																	> >			
11	17	41	41	0																	> >		> >	
12	15	23	28	5					>	2			>	6							>	5	>	7
13	15	28	28	0									> >										> >	
	16	28	32	4	>	8							>	4							>	5	> >	
14	16	32	32	0	> >																> >		> >	
	17	32	41	9	>	5															> >		> >	
15	17	34	41	7									>	3					>	0	> >		> >	
16	18	42	46	4	>	0															> >		> >	
17	18	46	46	0																	> >		> >	

Figure 20A

NETWORK INTERACTION LIMIT DETERMINATION - SHEET 2

Cycle →					11		12		13		14		15		16		17							
Operations Lengthened →					2,3 4,7		2,3 7		3,5 10		3,5 16		16 17		17		3							
Interaction Limit →					none		none		none		5		none		none		4							
Days Lengthened →						2		2		3		1		1		1		2						
Pre Op	Post Op	Pre EF	Post ES	Log		Log		Log		Log		Log		Log		Log		Log						
1	5			0																				
	6			0																				
2	10			0	> >		> >																	
3	13			0	> >		> >		> >		> >						> >							
4	14			0	> >		>	2	>	5														
5	7			0					> >		> >													
	12			5	>	7	>	9	> >		>	8												
6	8			0																				
	9			0																				
	10			0	>	2	>	4																
7	11			0	> >		> >		> >		> >													
8	11			14	>	16	>	18	>	21	>	22												
9	12			3	>	5	>	7	>	10														
10	12			0	> >		> >		> >															
	13			2	> >		> >		> >		>	3					>	5						
	14			0	> >		> >		> >															
11	17			0	> >		> >		> >		> >						>	2						
12	15			7	> >		> >		> >		>	8					>	10						
13	15			0	> >		> >		> >		> >						> >							
	16			5	> >		> >		> >		>	4					>	2						
14	16			0	> >		> >		> >															
	17			5	> >		> >		> >		>	6					>	8						
15	17			0	> >		> >		> >		> >						> >							
16	18			0	> >		> >		> >		> >		> >		>	1	>	3						
17	18			0	> >		> >		> >		> >		> >		> >		> >							

Figure 20B

PRECEDENCE MATRIX
Start @ All Crash

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1					O	O	X	X	X		X						X	X		
2										O		X		X		X				
3													O		X					
4																				
5							O				X						X	X		
6								O	O											
7											O						X	X		
8																				
9																				
10												O		O		X				
11																	O	X		
12																				
13															O					
14																O				
15																				
16																				
17																		O		
18																				
19																				
20																				

Figure 21

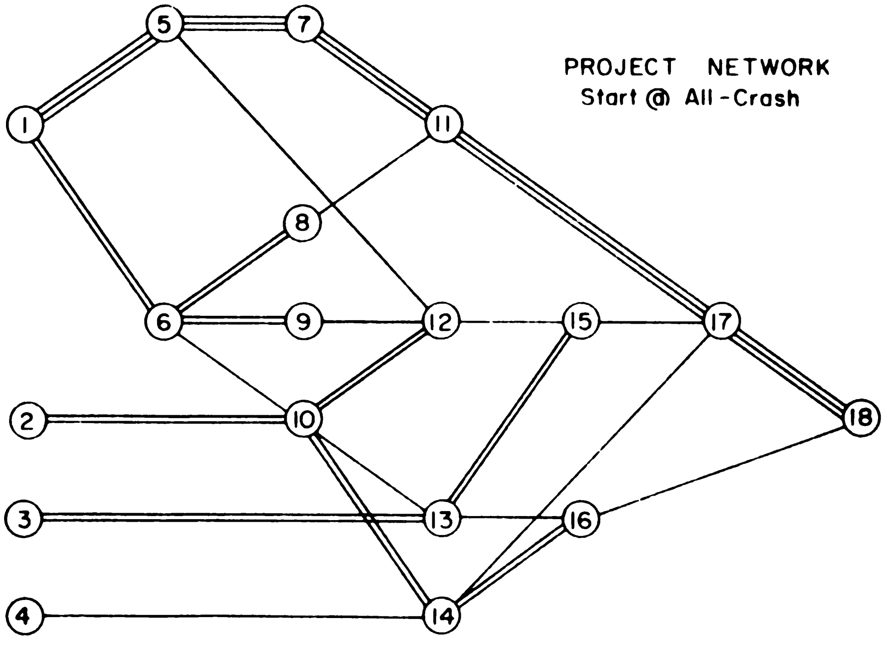

PROJECT NETWORK
Start @ All-Crash

Figure 22

DETERMINATION OF MAXIMUM OPERATION SLOPES COMBINATION

Operation Sequence	Cycle 1		Cycle 2		Cycle 6		Cycle 8		Cycle 9	
1 - 4	~~1~~	~~②~~	②	~~1~~	②	~~④~~	~~1~~	③	③	②
5 - 6	~~5~~	~~–~~	–	~~⑥~~	–	–	~~⑥~~	–	–	–
7 - 10	~~⑦~~	~~⑩~~			10	–	~~⑦~~	–	–	10
11 - 14	~~⑪~~	~~⑭~~					~~⑪~~	~~13~~		
15 - 16	~~–~~	~~⑯~~					–	~~⑮~~		
17	⑰	–								
18	Out-Cy.16		Out-Cy.11		Out-Cy.12		Out-Cy.17		Out-Cy.10	

Op.	Slope	Cycle 9								Cycle 10							11	12	13			14		15		16		17
14	400	X	X			X	X		X	X	X			X		X												
11	360	X		X	X		X	X		X		X	X		X													
9	300																											
12	300																											
7	250		X								X						X	X										
6	200			X																								
13	180																											
2	160			X								X					X	X										
10	150				X								X						X	X								
4	100			X	X							X					X											
8	80																											
17	70					X								X					X		X	X		X		X		
15	60	X	X	X	X			X	X																			
3	50						X			X	X	X	X		X	X	X	X		X			X		X		X	X
16	50							X							X						X	X	X	X				
5	40								X							X				X			X					
		820	×	(880)	670	470	×	×	×	(810)	×	670	×	470	×	×	(560)	(460)	220	(240)	×	120	(140)	(120)	50	(70)	×	㊿

Figure 23

APPENDIX F

PHASE III

"CONVENTIONAL-ESTIMATE START" PROCEDURE

TIME AND COST DATA – SLOPE DETERMINATION

Op.	Est Time	Est Cost	Crash Time	Crash Cost	Difference		Slope to Shorten	Normal Time	Least Cost	Difference		Slope to Lengthen	Earliest Start	Earliest Finish
					Time	Cost				Time	Cost			
1	5	1500	5	1500	0	0	none	5	1500	0	0	none	0	5
2	12	7680	10	8000	2	320	160	15	7200	3	480	160	0	12
3	25	8650	18	9000	7	350	50	30	8400	5	250	50	0	25
4	16	2500	14	2700	2	200	100	20	2100	4	400	100	0	16
5	10	1480	8	1560	2	80	40	12	1400	2	80	40	5	15
6	5	1000	4	1200	1	200	200	6	800	1	200	200	5	10
7	22	7300	20	7800	2	500	250	24	6800	2	500	250	15	37
8	6	1160	5	1240	1	80	80	8	1000	2	160	80	10	16
9	4	600	3	900	1	300	300	4	600	0	0	none	10	14
10	7	3450	7	3450	0	0	none	10	3000	3	450	150	12	19
11	10	2860	8	3580	2	720	360	11	2500	1	360	360	37	47
12	9	1800	6	2700	3	900	300	9	1800	0	0	none	19	28
13	12	2960	10	3320	2	360	180	14	2600	2	360	180	25	37
14	18	9600	15	10,800	3	1200	400	21	8400	3	1200	400	19	37
15	6	2140	6	2140	0	0	none	10	1900	4	240	60	37	43
16	10	1400	10	1400	0	0	none	12	1300	2	100	50	37	47
17	6	770	5	840	1	70	70	7	700	1	70	70	47	53
18	3	500	3	500	0	0	none	3	500	0	0	none	53	56
		57,350		62,630					52,500					

Figure 24

SUMMARY SHEET – PROJECT SCHEDULING ADJUSTMENTS

Cycle	Max.-Min Op. Slope Comb.				Operation Time Limits				Interact. Limit		Days Change	Cost Day	Cost Change	L or S C or NC	Total Job Cost	Total Job Days
	# 1	# 2	# 3	# 4	# 1	# 2	# 3	# 4								
0															57,350	56
1	14				3				6		3	400	1200	LNC	56,150	56
2	6				1				2		1	200	200	LNC	55,950	56
3	13				2				3		2	180	360	LNC	55,590	56
4	2				3				3		3	160	480	LNC	55,110	56
5	4				4				6		4	100	400	LNC	54,710	56
6	8				2				20		2	80	160	LNC	54,550	56
7	15				4				2		2	60	120	LNC	54,430	56
8	10	11	15		3	1	2		3		1	570	570	LC	53,860	57
9	10	17			1	1			3		1	220	220	SC	54,080	56
10	7	10	15		2	3	1		3		1	460	460	LC	53,620	57
11	3	5	10		7	2	1		3		1	240	240	SC	53,860	56
12	3	7	10		6	1	3		2		1	450	450	LC	53,410	57
13	3	5	10		7	1	1		1		1	240	240	SC	53,650	56
14	3	5	10									240		LC		

Figure 25

OPERATION SELECTION AND TIME TALLY SHEET

Op.	Slope $/Day	C. P. Cycle	Finish Cycle		Est. Time	Poss Short	Poss Lengt	Rev. Time / Remain. Poss. Short / Remain. Poss. Lengthening														
								1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
5	40	0	S 13 L		10	2	2											9 1 3		8 0 4		
3	50	7	S L		25	7	5											24 6 6	25 7 5	24 6 6		
16	50	4	S 0 L		10	0	2															
15	60	7	S 0 L 10		6	0	4							8 2 2	9 3 1		10 4 0					
17	70	0	S 9 L		6	1	1									5 0 2						
8	80		S L 6		6	1	2						8 3 0									
4	100	13	S L 5		16	2	4					20 6 0										
10	150	4	S 0 13 L		7	0	3								8 1 2	7 0 3	8 1 2	7 0 3	8 1 2	7 0 3		
2	160	4	S L 4		12	2	3				15 5 0											
13	180	7	S L 3		12	2	2			14 4 0												
6	200		S L 2		5	1	1		6 2 0													
7	250	0	S L 12		22	2	2										23 3 1		24 4 0			
9	300		S L 0		4	1	0															
12	300		S L 0		9	3	0															
11	360	0	S L 9		10	2	1									11 3 0						
14	400	4	S L 1		18	3	3	21 6 0														
1	none	0	S 0 L 0		5	0	0															
18	none	0	S 0 L 0		3	0	0															

Figure 26

NETWORK INTERACTION LIMIT DETERMINATION

Cycle →					1		2		3		4		5		6		7		8		9		10		11		12		13		14	
Operations Changed →					14		6		13		2		4		8		15		10,11 15		10,17		7,10 15		3,5 10		3,7 10		3,5 10			
Shortened or Lengthened →					L		L		L		L		L		L		L		L		S		L		S		L		S			
Interaction Limit →					6		2		3		3		6		20		2		3		3		3		3		2		1			
Days Changed →						3		1		2		3		4		2		2		1		1		1		1		1		1		
Pre Op	Post Op	Pre E.F.	Post E.S.	Lag		Lag		Lag		Lag		Lag		Lag		Lag		Lag		Lag		Lag		Lag		Lag		Lag		Lag		Lag
1	5	5	5	0																												
	6	5	5	0																												
2	10	12	12	0							>>																					
3	13	25	25	0																					<<		<<		<<			
4	14	16	19	3							>	6	>	2					>	3	<	2	>	3	<	2	<	1	<	0		
5	7	15	15	0																					<<				<<			
	12	15	19	4							>	7							>	8	<	7	>	8	<<		<	7	<<			
6	8	10	10	0			>>																									
	9	10	10	0			>>																									
	10	10	12	2			>	1			>	4																				
7	11	37	37	0																			>>		<<		<<		<<			
8	11	16	37	21			>	20							>	18							>	19	<	18	<	17	<	16		
9	12	14	19	5			>	4			>	7							>	8	<	7	>	8	<	7	<	6	<	5		
10	12	19	19	0							>>								>>		<<		>>		<<		<<		<<			
	13	19	25	6							>	3							>	2	<	3	>	2	<<		<<		<<			
	14	19	19	0							>>								>>		<<		>>		<<		<<		<<			
11	17	47	47	0															>>				>>		<<		<<		<<			
12	15	28	37	9					>	11	>	8							>	7	<	8	>	7	<<		<<		<<			
13	15	37	37	0					>>																<<		<<		<<			
	16	37	37	0	>	3			>	1	>	4							>	5	<	4	>	5	<<		<<		<<			
14	16	37	37	0	>>						>>								>>		<<		>>		<<		<<		<<			
	17	37	47	10	>	7					>	4							>>		<	5	>>		<<		<<		<<			
15	17	43	47	4					>	2							>	0	>>				>>		<<		<<		<<			
16	18	47	53	6	>	3					>	0							>>		<<		>>		<<		<<		<<			
17	18	53	53	0															>>		<<		>>		<<		<<		<<			

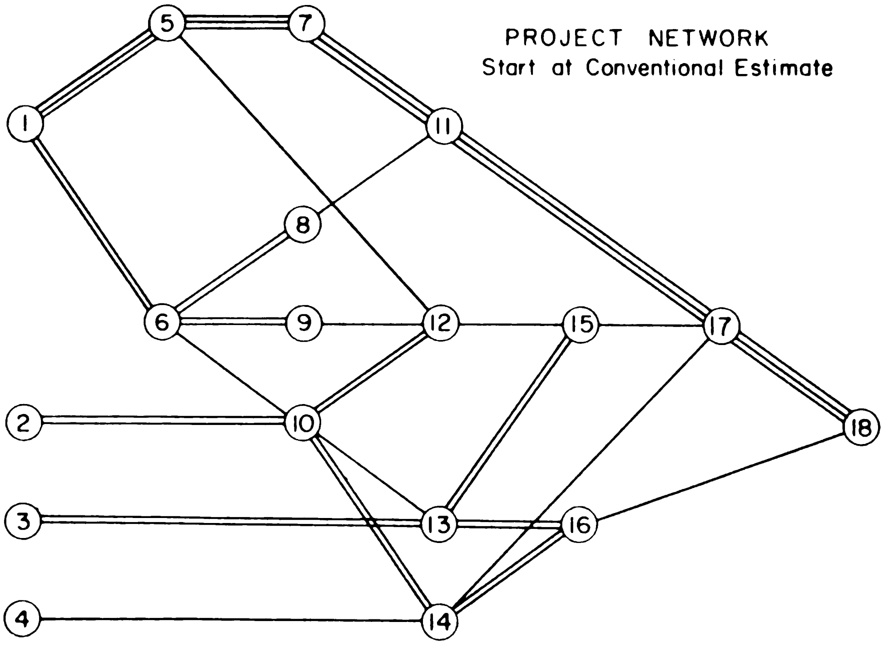
Figure 27

PRECEDENCE MATRIX

Start @ Conventional Estimate

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1					○	○	X	X	X		X						X	X		
2										○		X		X		X				
3													○		X	X				
4																				
5							○				X						X	X		
6								○	○											
7											○						X	X		
8																				
9																				
10												○		○		X				
11																	○	X		
12																				
13															○	○				
14																○				
15																				
16																				
17																		○		
18																				
19																				
20																				

Figure 28



PROJECT NETWORK

Start at Conventional Estimate

Figure 29

DETERMINATION OF OPERATION SLOPES COMBINATION

	Op. Sequence	Cycle 4		Cycle 7		Cycle 13	
Lengthening	1 - 4	~~1~~	~~②~~	~~1~~	③	~~2~~	~~4~~
	5 - 6	5	–	⑤	–	–	–
	7 - 10	~~⑦~~	⑩	~~⑦~~	–	⑩	–
	11 - 14	~~⑪~~	~~⑭~~	~~⑪~~	~~⑬~~		
	15 - 16	–	16	–	~~⑮~~		
	17	⑰	–				
	18						

Op.	Slope	8 L	Cycle 9 Short.					10 L	Cycle 11 Short				12 L	13 S	14 L		
5	40		X	X		X	X		X	X	X	X		X			X
3	50		X			X			X		X		X	X			X
16	50															X	
15	60	X		X				X		X							
17	70				X										X	X	
8	80																
4	100																
10	150	X	X	X	X		X	X	X	X		X	X	X	X		X
2	160					X					X						
13	180						X					X					
6	200																
7	250							X					X				
9	300																
12	300																
11	360	X															
14	400																
		$\boxed{570}$	240	X	$\boxed{220}$	X	X	$\boxed{460}$	$\boxed{240}$	X	X	X	$\boxed{450}$	$\boxed{240}$	220	X	$\boxed{240}$

	Op. Sequence	Cycle 4		Cycle 7		Cycle 13	
Shortening	1 - 4	1	2	1	③	2	④
	5 - 6	⑤	–	⑤	–	–	–
	7 - 10	7	⑩	7	–	⑩	–
	11 - 14	11	14	11	13		
	15 - 16	–	~~⑯~~	–	15		
	17	17	–				
	18						

Figure 30

APPENDIX G

OPTIMUM SOLUTION EXCEPTION

OPTIMUM SOLUTION EXCEPTION

A situation can arise where the manual procedures proposed by this report for the Phase III scheduling variations will not produce the optimum solution. Fortunately the probability that such a situation will occur is small and, if it should occur, the variation from the best possible solution is generally small. Therefore this shortcoming does not outweigh the advantages of providing a simplified approach that permits a manual solution to a complex problem.

When scheduling variations are made according to procedures proposed in this report, the operations changed in any one cycle are either all shortened or all lengthened. Consideration is not given to the possibility of concurrently shortening some operations while lengthening others. However a simple illustration will indicate a situation that could conceivably occur where concurrent shortening and lengthening gives a better solution.

Figure 31 shows a portion of a project network chart. For the purpose of illustration assume that these are the only operations in the network and that their performance times and cost slopes are as stated on the figure. If the project is being performed at least cost and normal time, the critical path consists of Operations 51-53-55 which require a project duration of 38 days. To shorten this duration, Operation 53 would be shortened by two days at a cost of $30 per day. At this point paths 51-54 and 52-55 both become critical, and Operation 53 has been shortened to its crash limit.

Further project duration shortening using the procedure proposed by this report would result in concurrently shortening both Operations 51 and 52 at a total cost of $110 per day. However a better solution is possible. Operations 51 and 55 can be concurrently shortened at a cost of $130 per day. Since both of these operations lie on critical path 51-53-55, that path is shortened two days for every one day that the other paths are shortened. This would cause it to become non-critical unless some operation on it that is not common to the other paths is lengthened. Operation 53, previously shortened at an expense of $30 per day, can be concurrently lengthened resulting in the recovery of $30 per day. The net result of the combination of simultaneously shortening Operations 51 and 55 and lengthening Operation 53 is to shorten project duration at a cost of $100 per day. This is a better solution than that giving $110 per day.

The existence of this situation may not be detected so easily in a complex network. Work is underway to determine if an improved, but still simple, procedure can be developed to give the optimum solution even when this special combination of network relationships, operation times and costs does occur. Note that the following conditions are required:

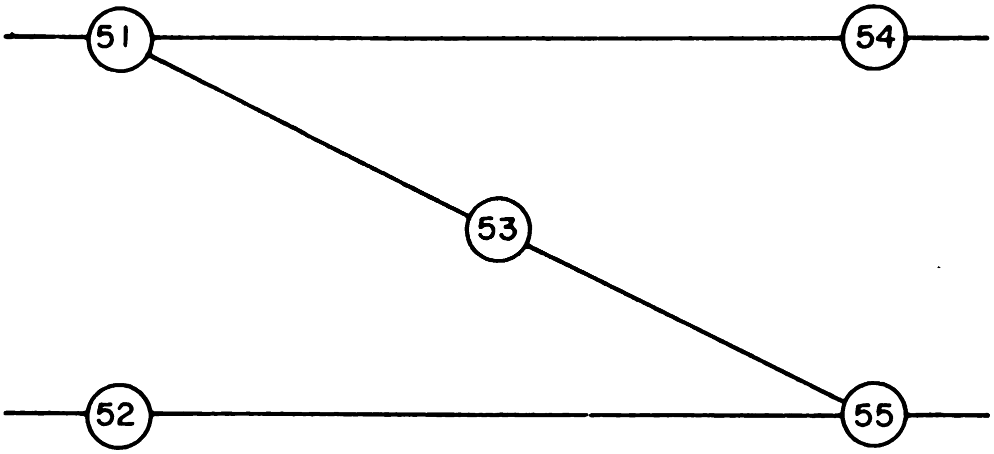
(1) There must be at least three critical paths.

(2) There must be some combination of operations common to all critical paths and more than one of these operations must be common to at least one of the critical paths.

(3) On a critical path that contains more than one of the operations being changed there must be an operation that is not common to any of the paths containing single operations being changed.

(4) It must be possible to change this operation, time-wise, in the opposite direction.

(5) The net result of simultaneously varying some operations in one direction and one or more in the opposite direction must be better than the result of varying the best combination of operations in one direction only.



PORTION OF PROJECT NETWORK CHART

TIME AND COST DATA			
Operation	Normal Time	Crash Time	Cost Slope $ / Day
51	8	4	50
52	16	10	60
53	10	8	30
54	28	20	70
55	20	15	80

FIG. 31

APPENDIX H

AN INFORMAL PRACTICAL APPROACH TO THE APPLICATION OF CPM

AN INFORMAL PRACTICAL APPROACH TO THE APPLICATION OF CPM

Further work on the use of CPM since the first edition of this report has strengthened the author's conviction that the application of the methods proposed is both practical and sound. It is the purpose of this Appendix to elaborate on two points which were previously given insufficient emphasis.

The first concerns the advantages of the circle and connecting line notation over arrow diagramming. The history of CPM indicates that early applications involved complex industrial plant problems having large numbers of operations. Those responsible for the development included mathematicians and operations research personnel. It is natural that a computer-oriented system was developed. This includes arrow diagramming which is now universally employed by available computer programs. The proposal of a different system for charting the project network should be based on a definite advantage to the contractor. There is such an advantage. With a minimum of instruction the man most familiar with the work may develop his own chart rather than relaying information on sequential relationships to a skilled specialist in CPM techniques who may understand little about the construction operations.

Consider the list of operations and sequential relationships shown on Figure 32. The operation numbers represent tasks that the supervisory personnel understand and for which there are sequencing relations, as shown, that they are best qualified to furnish. Now first attempt to draw a circle and connecting line network and next an arrow diagram to chart a model of this job. This problem was recently given to a group of contractors' personnel, many of whom were employing CPM in their work. In a couple of minutes, all were able to draw a correct network using the circle and connecting line notation. In a considerably longer time, none were able to develop the correct arrow diagram. Solutions are shown on Figures 3 and 4 of this report. The same problem was given to a group of 42 graduate engineering students as an outside assignment permitting them to take as much time as needed. Only four submitted a correct arrow diagram.

If a contractor's own personnel can easily draw network charts for their projects they can also easily understand them and will accept them more readily. If it then appears desirable to convert them to arrow diagrams in order that existing computer programs may be utilized, then a specialist, trained in these manipulations, can perform this task. The value of an easily constructed network chart, lacking dummy arrows, and being simple to revise, because arrow heads and tails do not have to be rigidly matched, should not be underestimated.

The second point that will be discussed here is a less formal but more practical approach to job planning and scheduling based on CPM techniques. The procedures proposed in this report for the important and difficult Phase III scheduling variations not only indicated the possibility of non-computer methods but also indicated certain advantages of these methods over the computer systems. However, the earlier discussion failed to stress that in using these non-computer methods, good judgment may considerably decrease the amount of effort required for sound planning and scheduling decisions. A less formal approach to be described below is not as satisfactory from a strictly theoretical mathematical standpoint because it does not require a complete set of time and cost input data to be furnished at the beginning of the solution process. Moreover it permits changes in the data at any point in the process on a judgment basis. For these same reasons that make this approach less satisfactory to the theorist, it becomes more useful to the competent planner and estimator.

The following suggested procedure utilizes a combination of CPM mechanics and sound judgment to gain the most from both:

(1) Draw a network chart that is reasonably accurate. Avoid too detailed an operation breakdown where it is recognized that overlapping of operations may be possible. Indicate the sequences that appear most realistic. A common problem here is whether to base sequential relationships on physical necessity or probable field scheduling restrictions. For example assume that on a building job after the building lines have been laid out, foundation excavation may be started on all of three different wings immediately since no other physical restrictions are present. However the planner is reasonably certain that the same crew or equipment will be used on each wing, and he therefore anticipates excavating for Wing A first, then for Wing B, and finally for Wing C. The question is whether the network

chart should be drawn based on this assumption or should it indicate that all three excavation operations can simultaneously follow the building surveying operation? There are two schools of thought on the proper choice. It is suggested here that the original network chart be drawn to indicate the excavation of the three wings as successive rather than concurrent, operations which is the most realistic sequencing in the planner's mind at this time.

(2) Having arrived at a network chart that appears to satisfactorily represent a realistic performance of the project, obtain time estimates for each operation. These time estimates need not be either the normal or crash estimates defined in the report. Rather they should represent reasonable estimates consistent with the time available for contract completion. These would be the times used for bar chart plotting under conventional planning procedures.

(3) Perform Phase II calculations to determine the critical operations, the floats of the non-critical operations, and the resulting project duration. Since this is a purely mechanical procedure, use an electronic computer if convenient. If not convenient, careful manual calculation can furnish this information within practical limitations of both time and effort.

(4) Knowing the critical operations, carefully examine them (and also operations having very small total floats indicating that they are nearly critical) to see if further breakdown is desirable. Some operations may have been shown as end-to-end tasks in the original network chart to avoid too detailed a breakdown. Now that it is known that these operations are critical ones, a closer examination is justified. By recognizing that some of these operations may be overlapped and should be broken down further, project time may be reduced from that first calculated.

(5) To the extent that further breakdown of operations is desirable to indicate that some may begin before others are 100% complete, add the resulting new operations and revise the original network. Steps (1) to (5) should be repeated as necessary.

(6) Next, consider the possibility of decreasing project time further without increasing project costs by reviewing just the critical operations. Generally these will only comprise perhaps 15 or 20 per cent of the total operations. With the knowledge that these are the operations that establish project duration, the planner can afford to take a closer look at them. Often it will be discovered that some of them can be performed in shorter times than originally estimated without changing their costs. There are several reasons that this is so. First, having recognized their importance the planner may devote more effort to arriving at faster ways to perform these operations. Second, with the knowledge of the non-critical operations in progress at the same time as any given critical operation, the planner may consider the possibilities of transferring men or equipment to expedite the critical operation. While the non-critical operation might be extended correspondingly, this would not affect project duration. Third, some of the critical operations may be performed by sub-contractors. In general, the sub-contractor has contractually agreed to adhere to the schedule established by the general contractor. Within the range of reasonable demands, he may be able to expedite his operations that are critical ones.

(7) Having made alterations in the project network chart and revisions to the time estimates of critical operations, Phase II calculations are again repeated. The resulting project duration may be either more or less than that specified by the owner. Changes in critical operations should now be made to bring the project duration in line with that specified. Here Phase III CPM mechanics can be profitably used. However a complete set of time and cost data for all operations

is not necessary. Only those operations that are critical, or that become critical because of the changes made, need be considered. Moreover, only the small portion of these that judgment tells the planner may possibly accomplish his aim need actually be considered. Therefore he generally will be dealing with only a handful of operations rather than a very large number. When critical operations must be shortened to reduce project time, the sequencing decisions made in step (1) should be reviewed. An economical means of shortening project duration may be to change the scheduling restriction that seemed reasonable at the outset. For example, at this time the planner may decide that the foundations of Wings A, B, and C should be excavated at the same time. The extra cost of additional labor or equipment may be the most economical way to speed project completion. This decision involves changes in the project network.

(8) Having made changes in the critical operations to bring the project duration to that required, further changes should be made to reduce project costs. This involves performance of the procedure described in the report section on "Phase III - Conventional Estimate Start." If necessary, Phase II calculations would be repeated once again to obtain new figures for the floats of the non-critical operations. Then the planner would carefully review the non-critical operations to see if any might be performed more economically in view of the fact that additional time is available. The fact that floats do exist indicates possible dips in manpower or equipment requirements if these non-critical operations are performed in the manner originally conceived. Recognizing that both labor and equipment may have to be carried over these dips for practical reasons, a review of methods is warranted. Possibly a smaller crew, for example, can do this non-critical job in the longer time permissible at a lower cost.

(9) Having exhausted possibilities of lowering costs by using more time for non-critical operations, the "wiggling-in" procedure for establishing a better balance between critical operations can be employed. Again it should be emphasized that a complete set of data is not required. The planner works with those critical operations that appear to give the results he is seeking and utilizes CPM mechanics to determine when important network relationship changes take place.

(10) Once the planner is satisfied that he has reached the most economical solution for the specified contract completion date, he may proceed to investigate the merits of other completion times. He may utilize CPM mechanics to develop a portion of the project time-cost curve on either side of the specified completion date. He may also develop an indirect cost curve over this range. Again, this need not be a complete curve, but need only indicate the incremental overhead costs associated with varying project completion over a relatively small range. If applicable the planner would consider liquidated-damage cost curves or bonus-penalty cost curves. The sum of all of these curves over just a portion of the time range representing all possible project durations should indicate a minimum total cost point. The corresponding project duration sets a better goal for the proposed project schedule than the completion date specified by the owner.

The foregoing procedure represents a practical approach to job planning and scheduling. It need not necessarily be carried through completely to produce important improvements in the original schedule. At no point does it require the estimator to furnish normal cost and time data and crash cost and time data for every operation as is required by the more formal computer solutions. It does permit frequent review of the time and cost data that is being used to determine if changes should be made. In summary, it takes advantage of the skills of the good planner and estimator while at the same time furnishing him a more powerful tool with which to work--CPM.

Network Sketching Exercise

Sketch network model using:

(a) Circle and connecting line notation

(b) Arrow notation

Network relationships are as stated below:

Operation	Must follow operation(s)
1	--
2	--
3	--
4	--
5	1
6	1
7	5
8	6
9	6
10	2, 6
11	7, 8
12	5, 9, 10
13	3, 10
14	4, 10
15	12, 13
16	13, 14
17	11, 14, 15
18	16, 17

Fig. 32